SIMG-782 Digital Image Processing

Homework 5

Due November 8, 2005

Ex. 1 — Define the function

$$f(x) = e^{-\pi x^2}$$

It can be shown that

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

(a) Show that

$$E_f = \int_{-\infty}^{\infty} f^2(x)dx = \frac{1}{\sqrt{2}}$$

(This is called the *energy* in $f(x)$.

(b) Show that $f(x)$ has the Fourier transform

$$F(u) = e^{-\pi u^2}$$

(c) Consider the periodic function

$$f_p(x) = \sum_{k=-\infty}^{\infty} f(x - kL)$$

Find an expression for the Fourier coefficients in

$$f_p(x) = \sum_{n=-\infty}^{\infty} c_n e^{i2\pi nx/L}$$

(d) Explore what happens when $f_p(x)$ is approximated by

$$f_p(x) = \sum_{n=-N_0+1}^{N_0-1} c_n e^{i2\pi nx/L}$$

with different choices for L and N_0.

(e) What does the sum

$$c_0^2 + 2\sum_{n=1}^{N_0-1} |c_n|^2$$

represent? What value do you expect it to approach for large L and large N_0?

Ex. 2 — We want to represent $f(x)$ of problem 1 by a list of sample values $f_n = f(nd)$, $n = -N, -N+1, \ldots, N-1, N$.

(a) Determine values for N and d such that the error

$$\mathcal{E} = \frac{1}{\sqrt{E_f}} \left| E_f - d \sum_{n=-N}^{N} f_n^2 \right| \leq 0.01$$

Relate the quantities N, W, L, d where W is a measure of the bandwidth and L is a measure of the duration.

(b) Calculate the DFT from the sample set that you choose. This will produce a set of Fourier coefficients. Compare the coefficient values to those you found in Problem 1.

Ex. 3 — Suppose that an image of dimensions 4×6 inches has detail to the frequency of 300 dots per inch in each direction.

(a) How many samples are required to preserve the information in the image?

(b) How many values are contained in the DFT $F(a, b)$ of the image?

(c) Suppose that the image is sampled at a frequency that corresponds to detail up to 600 dots per inch (but in reality the detail only goes to 300 dots per inch). What is the effect on the DFT?

Ex. 4 — Here we are going to examine filtering of an image in the frequency domain. Obtain the image 'barb.png' from the images directory. Investigate the effect of filtering the image with a Butterworth filter with parameter $p = 2$ and $D_0 = 10$, 20, 40 and 80. (See lecture 12, page 20). Explain the results in terms of both frequency domain and spatial domain concepts.