

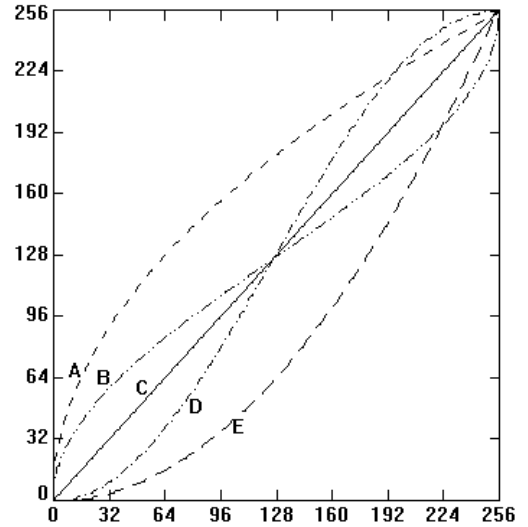
SIMG-782 Introduction to Digital Image Processing

Midterm Examination Fall Quarter 2003

Closed book and notes. Calculators are approved. Answer all questions.

1. The diagram at the right contains several curves that could be used to transform the brightness values of a monochrome image by the operation $B = T[A]$ where A and B are image arrays. Shown below are four pairs of histograms. Identify the transformation curve best associated with each pair and write the letter in the space in the center column. *Curve C is the identity transform.*

Curves A and B stretch the dark regions while curves D and E compress the dark regions. Curve B stretches the light region while curve D compresses the light region. Compare these effects with the changes in the histograms to select the transforms. Answers indicated below.



Input image histogram	Transform	Output image histogram
	E	
	D	
	A	
	B	

2. The Sobel operator computes the following quantity at each location (x, y) in an image array, A :

$$\begin{aligned} G_x[j, k] &= (A[j + 1, k + 1] + 2A[j + 1, k] + A[j + 1, k - 1]) - (A[j - 1, k + 1] + 2A[j - 1, k] + A[j - 1, k - 1]) \\ G_y[j, k] &= (A[j - 1, k - 1] + 2A[j, k - 1] + A[j + 1, k - 1]) - (A[j - 1, k + 1] + 2A[j, k + 1] + A[j + 1, k + 1]) \\ G[j, k] &= |G_x[j, k]| + |G_y[j, k]| \end{aligned}$$

The position of $A[j, k]$ is column j and row k of the array.

The operation is implemented as the convolution of the image array A with two masks, M_x and M_y followed by the magnitude operation.

(a) **Write a 3×3 array for each mask, M_x and M_y .**

The equations represent the formation of sums along rows or columns that are adjacent to pixel $[x, y]$ and taking the difference. This can be achieved by convolving with the masks below. True convolution would reflect the masks about the origin while CONVOL does not do so. The reflection is not important here.

$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(b) **What mathematical operation on an image array is approximated by the Sobel operator? Show how the Sobel operator is related to the mathematical operation.**

$G = |G_x| + |G_y|$ approximates the magnitude of the gradient. The gradient vector is given by

$$\nabla A = \begin{bmatrix} \frac{\partial A}{\partial x} \\ \frac{\partial A}{\partial y} \end{bmatrix} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

The magnitude of the gradient is

$$|\nabla A| = [A_x^2 + A_y^2]^{1/2} \approx |A_x| + |A_y|$$

These can be approximated by the finite differences

$$\begin{aligned} A_x &\approx A[x + 1, y] - A[x - 1, y] \\ A_y &\approx A[x, y + 1] - A[x, y - 1] \end{aligned}$$

G_x uses the row above and below the current position with the weight 2 in the central position to give more emphasis to pixels adjacent to $[x, y]$. The same applies to G_y with the words 'row' and 'column' exchanged.

3. Answer the following questions about morphological image processing.

- (a) Shown below are two tables with expressions that relate to binary morphological image processing. Associate each expression in the left table with one from the right table.

a	$A - B$
b	\hat{B}
c	$A \oplus B$
d	$A \ominus B$
e	$A \circ B$
f	$A \bullet B$

1	$\{w w = -b, \text{ for } b \in B\}$
2	$\{z (\hat{B})_z \cap A \neq \emptyset\}$
3	$(A \oplus B) \ominus B$
4	$A \cap B^c$
5	$(A \ominus B) \oplus B$
6	$\{z (B)_z \subseteq A\}$

Answers					
a	b	c	d	e	f
4	1	6	2	5	3

- (b) A well-known morphological algorithm uses the following iteration with a structuring element B .

- (1) Initialize $X[p] = 1$ for some pixel $p \in A$
- (2) $Y = (X \oplus B) \cap A$
- (3) If $Y \neq X$ then set $X = Y$ and repeat (2)

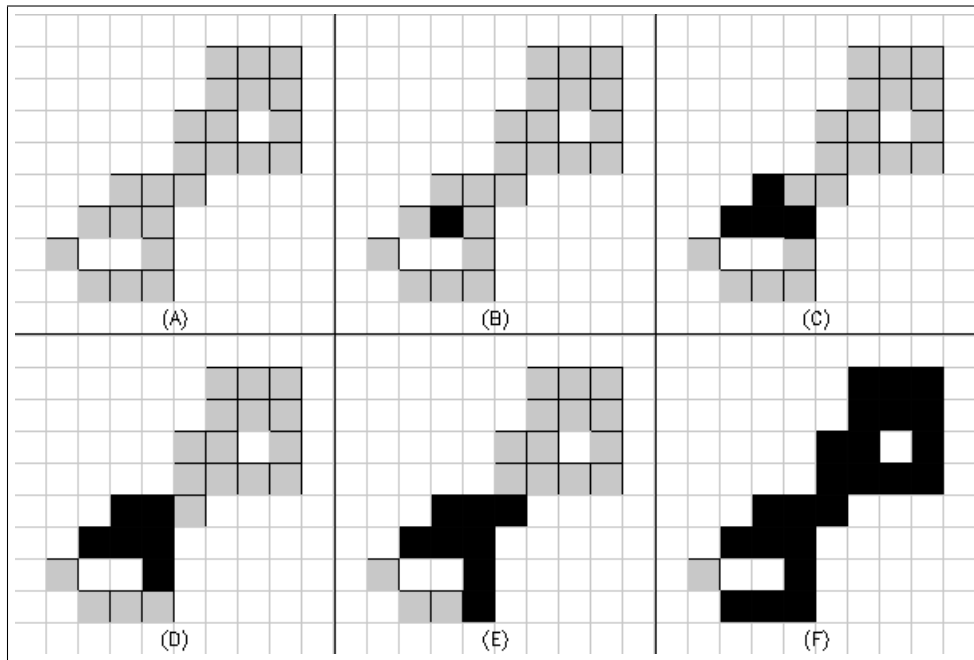
An original set A is shown in (A) and an initial pixel $p \in A$ is shown in (B). The result after one iteration of the algorithm with structuring element

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

is shown in (C).

Fill in the result of the next two iterations by marking the appropriate pixels for the set Y in (D) and (E).

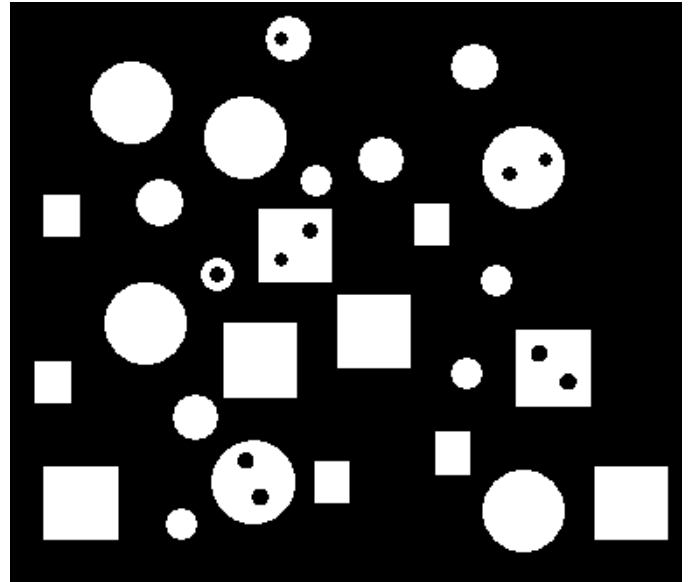
In frame (F) show the result for Y that would be reached after a large number of iterations.



4. A certain inspection application gathers black & white images of parts as they travel along a conveyor belt. It is necessary to sort the parts into two categories: parts with holes and parts without holes. An example of an image that might be taken by the inspection camera is shown at the right.

Propose a method to identify and locate the objects of each category in the image so that they can be picked up by a robotic system and placed in different bins. Assume that the imaging system knows where each image pixel is located on the conveyor belt at every point in time.

Provide an annotated flow chart of the algorithm you propose.



A number of algorithms are possible here. One such is given below.

Call the input image A .

Step 1: Locate the background pixels by performing `Label_Region` on A^c . The background B is the region with the greatest number of pixels.

Step 2: Complement the background. B^c is an image with all objects filled.

Step 3: Perform `Label_Region` on B^c .

Step 4: For each object identified in step 3:

(a) Construct an image A_k with the dimensions of A that contains only filled object k .

(b) Find the centroid of A_k .

(c) Compare that object to image A . If $A \cap A_k = A_k$ then A_k has no holes, otherwise A_k has one or more holes.

The centroid of an object A_k is found by

$$x_k = \frac{\text{TOTAL}(A * X)}{\text{TOTAL}(A)}$$

$$y_k = \frac{\text{TOTAL}(A * Y)}{\text{TOTAL}(A)}$$

where X and Y are coordinate arrays that match the image dimensions.

5. An image array A and associated color vectors is acquired from a file `fname` and displayed in color with the following commands: (You may use IDL statements in your answer or otherwise explain how to construct the results.)

```
A=Read_Image(fname,rr,gg,bb)
sa=Size(A,/dimensions)
Window,1,xsize=sa[0],ysize=sa[1]
Device,Decomposed=0
TVLCT,rr,gg,bb
TV,A
```

- (a) A person wants to see the image as a grayscale display, and does the following:

```
Loadct,0 ;Load the BW Linear Palette
TV,A ;Display the image
```

The result is all blotchy and bad looking. Explain what is wrong and devise a method to use the available information to construct an array B that will display the image correctly in grayscale when used in the following:

```
Device,Decomposed=0
Loadct,0
TV,B
```

A is the color index array, and does not directly contain brightness information. To get a brightness we form a weighted average of the red, green and blue color planes. If we use equal weights then:

$$B = \text{Byte} \left[\frac{1}{3} (\text{Float}(rr[A]) + \text{Float}(gg[A]) + \text{Float}(bb[A])) \right]$$

- (b) Another person wants to display the image in color using a true color display. Devise a method to use the available information to construct an array C that will display correctly in true color when the following commands are issued.

```
Device,Decomposed=1; Put the display in the true color mode
TV,C,True=1 ;Display in true color. Note the True=1 direction
```

We want an array that is three planes deep.

```
C=BytArr(3,Sa[0],Sa[1] ;Construct an empty array
C[0,*]=rr[A] ;Red plane
C[1,*]=gg[A] ;Green plane
C[2,*]=bb[A] ;Blue plane
```