1. The diagram at the right contains several curves that could be used to transform the brightness values of a monochrome image by the operation $B = T[A]$ where $A$ and $B$ are image arrays. Shown below are four pairs of histograms. Identify the transformation curve best associated with each pair and write the letter in the space in the center column.
2. The Sobel operator computes the following quantity at each location \((x, y)\) in an image array, \(A\):

\[
\]

\[
\]

\[
\]

The position of \(A[j, k]\) is column \(j\) and row \(k\) of the array.

The operation is implemented as the convolution of the image array \(A\) with two masks, \(M_x\) and \(M_y\) followed by the magnitude operation.

(a) Write a 3 \(\times\) 3 array for each mask, \(M_x\) and \(M_y\).

(b) What mathematical operation on an image array is approximated by the Sobel operator? Show how the Sobel operator is related to the mathematical operation.
3. Answer the following questions about morphological image processing.

(a) Shown below are two tables with expressions that relate to binary morphological image processing. Associate each expression in the left table with one from the right table.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$A - B$</td>
<td>1</td>
<td>${w \mid w = -b, \text{for } b \in B}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>$B$</td>
<td>2</td>
<td>$B_2 \cap A \neq \emptyset$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>$A \oplus B$</td>
<td>3</td>
<td>$(A \oplus B) \ominus B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>$A \ominus B$</td>
<td>4</td>
<td>$A \cap B^c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>$A \circ B$</td>
<td>5</td>
<td>$(A \ominus B) \oplus B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>$A \bullet B$</td>
<td>6</td>
<td>${z(B)_2 \subseteq A}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) A well-known morphological algorithm uses the following iteration with a structuring element $B$.

1. Initialize $X[p] = 1$ for some pixel $p \in A$
2. $Y = (X \oplus B) \cap A$
3. If $Y \neq X$ then set $X = Y$ and repeat (2)

An original set $A$ is shown in (A) and an initial pixel $p \in A$ is shown in (B). The result after one iteration of the algorithm with structuring element

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

is shown in (C).

Fill in the result of the next two iterations by marking the appropriate pixels for the set $Y$ in (D) and (E).

In frame (F) show the result for $Y$ that would be reached after a large number of iterations.
4. A certain inspection application gathers black & white images of parts as they travel along a conveyor belt. It is necessary to sort the parts into two categories: parts with holes and parts without holes. An example of an image that might be taken by the inspection camera is shown at the right.

Propose a method to identify and locate the objects of each category in the image so that they can be picked up by a robotic system and placed in different bins. Assume that the imaging system knows where each image pixel is located on the conveyor belt at every point in time.

Provide an annotated flow chart of the algorithm you propose.
5. An image array $A$ and associated color vectors is acquired from a file $\text{fname}$ and displayed in color with the following commands: (You may use IDL statements in your answer or otherwise explain how to construct the results.)

```idl
A=Read_Image(fname,rr,gg,bb)
sa=Size(A,/dimensions)
Window,1,xsize=sa[0],ysize=sa[1]
Device,Decomposed=0
TVLCT,rr,gg,bb
TV,A
```

(a) A person wants to see the image as a grayscale display, and does the following:

```idl
Loadct,0 ;Load the BW Linear Palette
TV,A ;Display the image
```

The result is all blotchy and bad looking. Explain what is wrong and devise a method to use the available information to construct an array $B$ that will display the image correctly in grayscale when used in the following:

```idl
Device,Decomposed=0
Loadct,0
TV,B
```

(b) Another person wants to display the image in color using a true color display. Devise a method to use the available information to construct an array $C$ that will display correctly in true color when the following commands are issued.

```idl
Device,Decomposed=1; Put the display in the true color mode
TV,C,True=1 ;Display in true color. Note the True=1 direction
```