1. An image $A$, represented by an $N \times M$ array of bytes, has a uniform brightness histogram. It is desired transform $A$ into an image $B$ in a way that produces a triangular brightness histogram

$$h_b[k] = \frac{MN}{36240} k, \ 0 \leq k \leq 255$$

Describe a process that will accomplish the transformation. If possible, derive an equation for the transformation function. At a minimum, sketch the transformation function and indicate how you would use it in a program to compute the array $B$. 
2. An indexed color image is represented by an 512×512 array $A$ of bytes and three color vectors, $r$, $g$ and $b$ of length 256, also of bytes.

(a) Design a computation that can be used to find the total number of colors that are actually used by the image. Describe your reasoning and provide a pseudo-code algorithm.

(b) Design a way to count the number of pixels in which the red fraction is less than $1/3$. For example, a pixel with the color mixture $[R|G|B] = [100|200|200]$ would have $100/500 = 20\%$ red, $200/500 = 40\%$ green and $200/500 = 40\%$ blue.
3. A preprocessing step in an application of microscopy is concerned with the issue of isolating individual round particles from similar particles that overlap in groups of two or more.

Assuming that all particles are of the same size, propose a morphological algorithm that will produce an image that contains only the isolated (non-overlapping) particles that are not in contact with the boundary of the image.
A CCD color TV camera is used to perform a long-term study by observing the same scene 24 hours a day for 30 days. Digital images are captured and transmitted to a center every 5 minutes. The illumination of the scene changes from natural daylight to artificial lighting, which alters both the brightness and color balance of the illumination. At no time is the scene without illumination, so it is always possible to obtain an image. Because the range of illumination is always in the linear operating range of the camera, it is decided to use digital techniques to postprocess, and thus normalize, the images to the equivalent of constant illumination.

Propose a method to normalize the images to maintain the equivalent of constant illumination. This means having both the correct brightness and the correct color balance. Clearly state any assumptions, and provide a description and block diagram of your design.
5. In a given application, an averaging mask is applied to input images to reduce noise and then a Laplacian mask is applied to enhance small details. Would mathematics predict that the result should be the same if the order of the operations were reversed? What practical issues would be encountered in computer implementation?