

Name: _____

SIMG-782 Introduction to Digital Image Processing

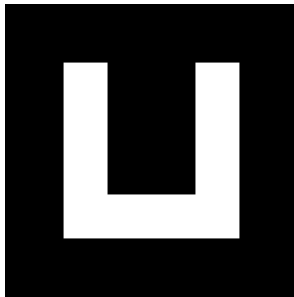
Final Exam

November 20, 2002

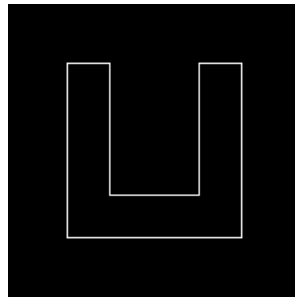
Closed book and notes. Work all problems. Clearly show your reasoning and identify the solution.

1. Shown below are a number of image pairs. In each case the image A was processed by a morphological transform to produce the image B . Your task is to specify the morphological operations that were used given the information provided.

(a) A structuring element S of size 3×3 was used in this transform. B satisfies $B \subset A$.



A

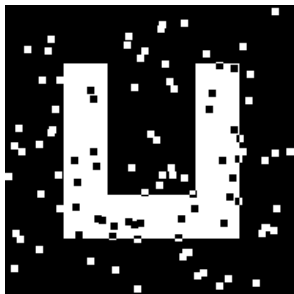


B

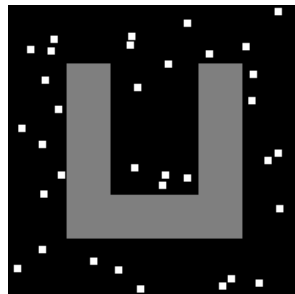
- (a) $B = (A \ominus S) \cap A^c$
- (b) $B = (A \oplus S)^c \cup A$
- (c) $B = (A \oplus S) \cap A^c$
- (d) $B = (A \ominus S)^c \cap A$

Answer

(b) Image B was produced from A by an operator that used two structuring elements, S_1 , which exactly matches one of the small square boxes, and S_2 , which fits snugly around the small square boxes. The gray image object in B is shown only for reference, and is actually not present. Determine which of the operations listed below was used in this transformation.



A



B

- (a) $B = [(A \ominus S_1) \cap (A^c \ominus S_2)] \ominus S_1$
- (b) $B = [(A \ominus S_1) \cap (A^c \ominus S_2)] \oplus S_1$
- (c) $B = [(A \oplus S_1) \cap (A^c \oplus S_2)] \cap A$
- (d) $B = [(A \ominus S_1) \cap (A^c \ominus S_2)] \cap A$

Answer

2. An image represented by a continuous function $f(x, y)$ is $w = 2$ cm wide and $h = 3$ cm high. The image is to be converted to an array of pixels by a scanner whose response is zero above 80 lines/centimeter in both the horizontal and vertical directions. The discrete image is represented by an array $\hat{f}(n, m)$ where n and m take on integer values, $0 \leq n \leq N - 1$, $0 \leq m \leq M - 1$.

(a) Determine suitable values for N and M .

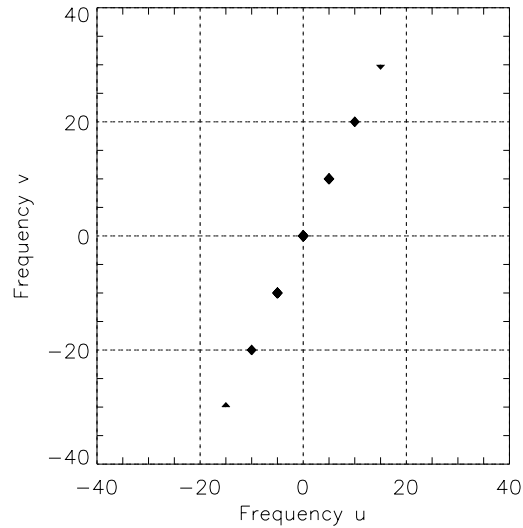
(b) Assume that $\hat{f}(n, m) = f(na, mb)$. Determine the values of a and b .

(c) Determine constants A, B, C, D, E such that the DFT of \hat{f} can be expressed as

$$F(u, v) = A \sum_{n=0}^B \sum_{m=0}^C \hat{f}(n, m) e^{-i(Dun + Evm)}$$

(d) Find numbers (P_1, P_2) such that $F(u + jP_1, v + kP_2) = F(u, v)$ for any integers j, k, u, v .

3. A certain detector has a 200×200 array of detector elements. It has been found that the images produced by the detector suffer from additive harmonic distortion, which can be represented by $q(x, y)$. A contour plot of the Fourier transform magnitude, $|Q(u, v)|$, is shown below for a central portion of the frequency domain.



- (a) Describe the main characteristics of the function $q(x, y)$.
- (b) Design process to suppress the harmonic distortion in images produced by this detector array. Provide values for the important system parameters. Describe how your system would work.

4. The gradient of a function $f(x)$ is defined as

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Computationally, the first derivative is implemented by calculating the difference between adjacent pixels.

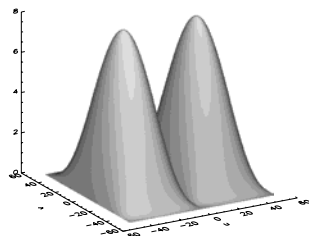
(a) Is the following a linear operator?

$$|\nabla f| = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{\frac{1}{2}}$$

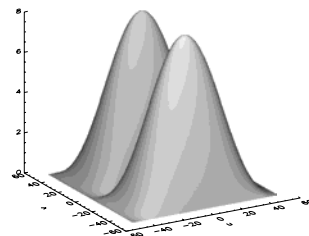
(b) State how would you implement the above operator using differences between pixels.

(c) A Sobel operator uses two masks, H_x and H_y to process an image. Explain why are two masks needed and what do they measure?

(d) Write down the masks H_x and H_y , and identify them in the following figures:

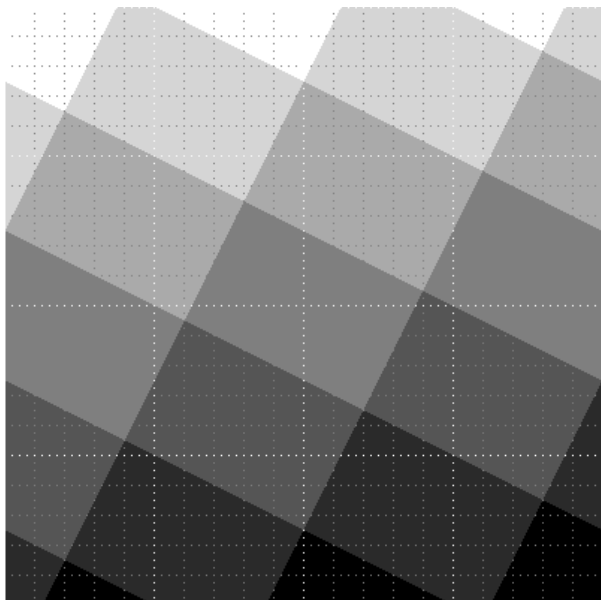


$|H_x(u, v)|$

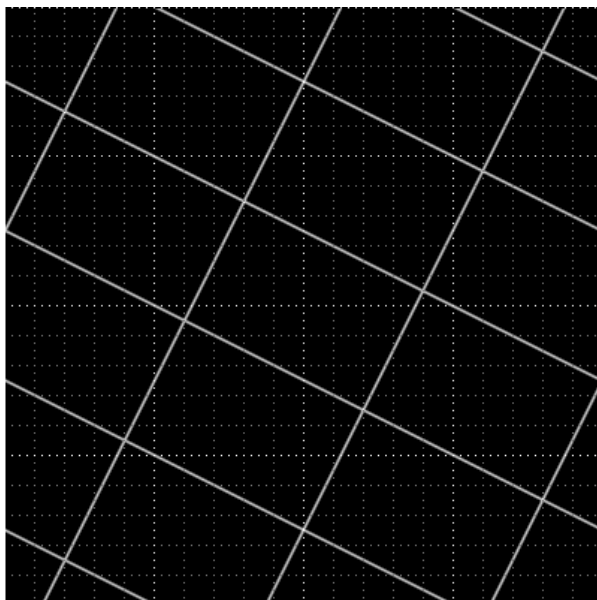


$|H_y(u, v)|$

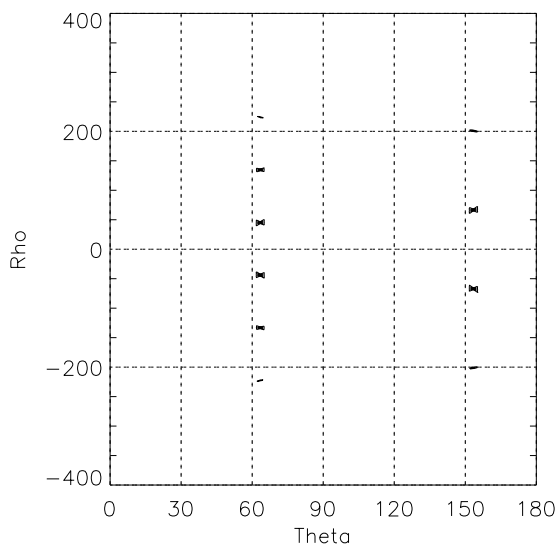
5. An image of a scene with a pattern of rectangles is shown in Figure (a). The image array is 400×400 pixels. A set of gridlines has been placed on top of the image. Assume that $(0,0)$ is at the center. Figure (b) is an image that was produced by using an edge detector on image (a), with the same set of gridlines imposed. Figures (c) and (d) show a surface plot and a contour plot, respectively, of a Hough transform that was produced from image (b). Derive a relationship between the lines in (a) and (b) and the major features of (c) and (d). Match corresponding lines and features. Provide numerical values to justify your result.



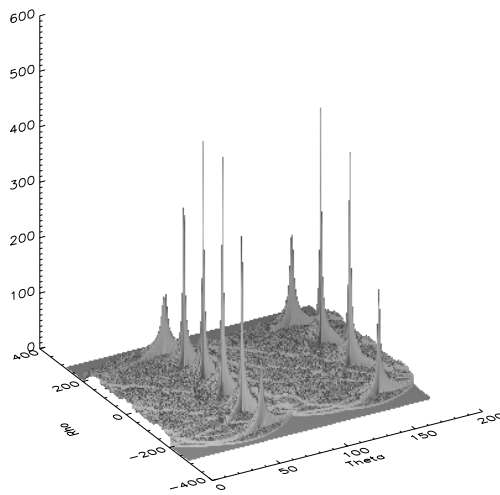
(a)



(b)



(c)



(d)