Review Questions

1. Describe the relationship between physical, mathematical and computational representations of images and the way they interact in the research and development of processing systems.

2. What is "signal space"? When we say that **f** and **g** are elements of signal space, what properties must they have? How can we treat images as elements of signal space?

3. Images can be represented in computers as arrays. Locations in arrays are determined by array coordinates ($A[x, y, z, \ldots]$) and array indexes ($A[p]$). How does one translate between them? What information do you need to do the translation, and how do you use it?

4. Describe two approaches for the representation of color images. How does the choice affect the ability to represent color information? What are the strengths and weaknesses of each approach?

5. Given an index color image

   (a) Give the algorithm steps to construct a true-color image.

   (b) Give the algorithm steps to construct the intensity (gray scale) image.

   (c) How does the gray scale image differ from the array values?

6. Given a true-color image

   (a) Give the algorithm steps to construct an index color image.

   (b) Give the algorithm steps to construct the intensity image.

7. Spatial structure in images is discussed with a number of concepts. How do these interact and what are some of the effects produced by different choices?

   (a) Neighborhoods

   (b) Paths

   (c) Connected components

   (d) Object connectivity (e.g. how many holes, etc.)

8. Describe the approaches to finding connected components in image arrays.

   (a) Region labeling, and its use with arrays in which objects can have different properties, such as different colors.

   (b) Morphological processing, in which all pixels are treated as having binary values.

   (c) How could a binary connected components algorithm be used to find objects with different colors in an image?

   (d) How could you use a CC algorithm to differentiate between solid objects and objects with holes?

9. Let A be an $N \times M$ 2D array and T a 1D vector with K elements. What must be the value of K so that the IDL expression $B = T[A]$ is meaningful? What data types can be used for A for this function array approach to work?

10. Describe the uses of $B = T[A]$ in image enhancement. For each case, describe how you would come up with the vector T. Some examples:

    (a) Gamma correction

    (b) Histogram equalization

    (c) Matching image grayscales

11. Given images A and B and assume that the background pixels have value 0. Say what each of the following statements mean and give the IDL statement to compute it. Give another IDL statement that can be used to find the pixels in the resulting object.

    (a) $C = A \cup B$

    (b) $C = A \cap B$

    (c) A^c

    (d) $A - B$

12. Describe the following morphological operations for binary images and give an example of their use in image processing:

    (a) Dilation

    (b) Erosion

    (c) Opening

    (d) Closing

13. Describe an algorithm and give the IDL statements to compute the boundary of an object in a binary image.

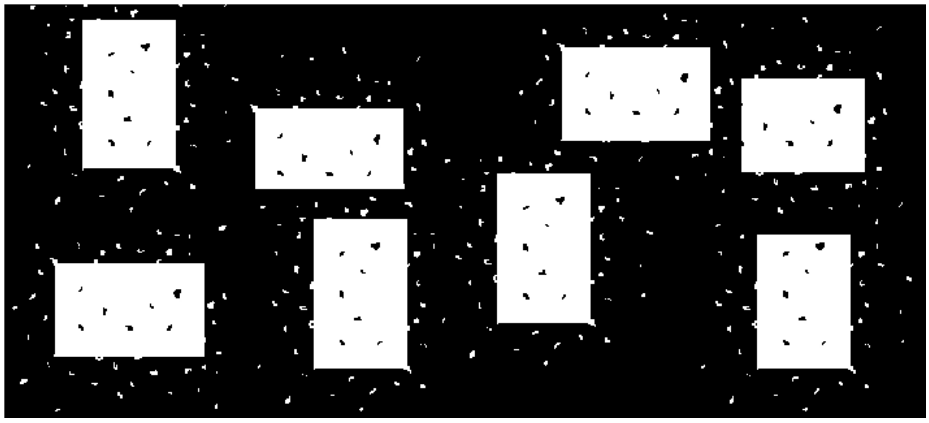
14. Describe an algorithm and give the IDL statements for filling the hole inside a binary object.

15. Describe the elements of the hit-or-miss transform.

16. Describe an algorithm and give the IDL statements for finding the convex hull of a binary object.

17. Describe an algorithm and give the IDL statements for thinning a binary object.

18. Describe an algorithm to locate all of the large boxes in the image below.



19. The mathematical operation on an image **f** to produce an image **g** represented by

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

    is the core element of linear spatially-invariant filtering. What is the corresponding statement in IDL and what are the data structures that are used?

20. Give the linear filter masks for the following operations:

    (a) Region averaging

(b) Weighted region averaging (with pixels near the center given more weight)

(c) Emphasizing pixels with large differences from the background

(d) Detecting vertical (horizontal) lines

(e) Detecting vertical (horizontal) edges

(f) Image sharpening

(g) Image sharpening while preserving a proportion of the original

(h) A feature detector for hollow boxes of size 5×5

21. What is a median filter and what is it used for?

22. Show that a median filter is not a linear filter.

23. What does the following mathematical statement mean, and how can it be implemented in a computer program? What are some of the options for its implementation? What is it used for?

$$|\nabla f| = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

24. What does the following mathematical statement mean, and how can it be implemented in a computer program? What are some of the options for its implementation? What is it used for?

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

25. A spatial filter can be described as a mask such as

$$W = \begin{bmatrix} w_0 & w_1 & w_2 \\ w_3 & w_4 & w_5 \\ w_6 & w_7 & w_8 \end{bmatrix}$$

Relate the operation of spatial filtering to processing by a linear system with an impulse response h.

26. An image F is to be processed by an operation that can be represented by spatial filtering with an array $W_3 = W_1 + W_2$. What assumptions do we make if we say that we can either operate with W_3 or sum the results of operating with W_1 and W_2 and get the same effect? Is there any risk of different answers if we are doing byte processes? What precautions are needed?

27. Suppose that $W_1 = I$ is an identity operator and W_2 is a lowpass operator. Explain how you would combine them to create a highpass operator HP.

28. What are the properties of a filter F given by the array below? The center element is $w = 9A-1$, where $A \geq 1$. This filter is called an unsharp mask and is commonly used in graphic arts and photography.

$$F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

29. A Sobel operator uses two masks, H_x and H_y to process the image. Explain what is measured and why two masks are needed.

30. Is a Sobel operator linear?

31. What would happen if one processed an image F with a Sobel operator to produce an image G and then combined the results with the IDL statement $H = F > G$? How could this idea be extended to produce a kind of "unsharp masking" effect?

Hough Transform

32. A Hough transform is to be used to search for curves of the form $ax^2by^2 = c^2$. What kind of surface in (a, b, c)-space represents every point on a particular curve?

33. Estimate the number of Hough cells that are required as a function of the number of parameters to be searched and the resolution that is needed in their values. Assume that each parameter must be determined to within some fraction f of its total range.

34. A line has the parametric description $2 = x\cos\pi/3+y\sin\pi/3$. What is the parametric equation for a line that is perpendicular to the given line and passes through the point $(5, 5)$?

35. A Hough diagram is constructed by using the parametric form $\rho = x\cos\theta + y\sin\theta$. Can you think of a procedure for searching for patterns of filled Hough cells that will reveal perpendicular line structures in the image?

36. Why does a practical linear Hough transform algorithm need parameters to limit line lengths and line gap sizes?

Fourier Transform and Sampling

37. What algorithm could be used to find the fraction of the "energy" in an image in a given band of frequencies, say $u_1 \leq u \leq u_2,\ v_1 \leq v \leq v_2$? How could this be employed to find the 50% bandwidth?

38. You are given a CCD array with 1000×1000 pixels and a good optical system that lets you zoom and focus on objects in a scene. Derive a relationship between the size of the image field and the smallest detail that you can capture with this system.

39. Suppose that you want to build a filter $H(u, v)$ that will respond to vertical stripes of width 128 on the 1000×1000 CCD array. What is the equation for such a filter? What would you do to make the system sensitive to a horizontal displacement of the stripes? What would you do to make it insensitive to a horizontal displacement?

40. An image has the Fourier transform $F(u, v)$. A new image $G(u, v)$ is created by the algorithm $G(u, v) = F(u, v)$ followed by setting $G(u_0, v_0) = 0$ and $G(-u_0, -v_0) = 0$. What is the difference $f(x, y) - g(x, y)$?

41. Given that a discrete function of length N samples has the DFT

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)e^{-i2\pi ux/N}$$

derive the expression

$$f(x) = \sum_{x=0}^{N-1} F(u)e^{i2\pi ux/N}$$

42. A function $f(x)$ defined on the interval $0 \leq x \leq L$ centimeters has a maximum frequency W cycles per centimeter. It has been found that the Fourier transform of $f(x)$ is approximated by

$$F(u) = \frac{1}{1 + (\frac{4u}{W})^2}$$

Write an expression for the Fourier series for the periodic extension of $f(x)$.

43. Provide a pseudo-code algorithm to convolve a 5×5 filter $h(x, y)$ with a $N \times M$ image $f(x, y)$ using frequency-domain multiplication. The result should be the same as $g(x, y) = f(x, y) \star h(x, y)$

44. Provide a pseudo-code algorithm to implement a filter whose mask is illustrated Figure 1. Assume that the filter has response $H(u, v) = 1$ in the white region and $H(u, v) = 0$ in the dark region. Given $F(u, v)$, your program should do the equivalent operation of $G(u, v) = H(u, v)F(u, v)$. The white bars in the figure are nine pixels in width.

Figure 1: Filter response for Problem 44

45. Describe the kind of distortion that is present in Figure 2 and provide a means to eliminate it.

Figure 2: Filter response for Problem 45

46. The function $f_P(x, y)$ is periodic with period (P_1, P_2). Show that $F(u, v)$ is also periodic with period (P_1, P_2).

$$F(u,v) = \frac{1}{P_1 P_2} \sum_{x=0}^{P_1-1} \sum_{y=0}^{P_2-1} f_P(x,y) e^{-i2\pi\left(\frac{xu}{P_1}+\frac{vy}{P_2}\right)}$$

47. The histogram for a 100×100 image $f(x, y)$ is described by the function

$$h(n) = \frac{n(256-n)}{279.616},\ 0 \le n \le 255$$

What is the value of

$$\sum_u \sum_v |F(u,v)|^2$$

for this image?