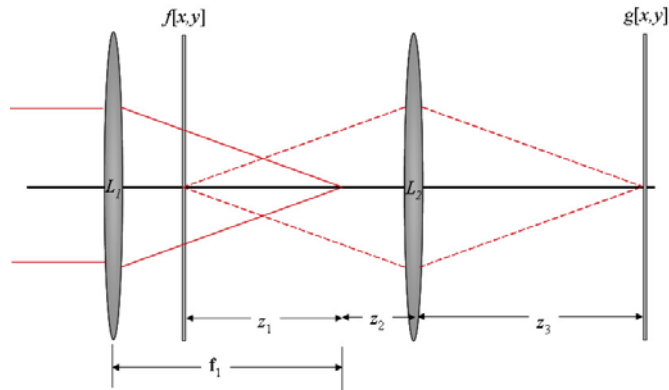


Select 4 of the 6 problems, may do either or both of the other 2 for extra credit (SPECIFY which)

1. Virtually all of the optical systems considered in this class consisted of a single optical element with the object distance z_1 and observation plane located at z_2 ; the only exceptions were the cascades of such systems to produce the so-called “4f” and “6f” optical correlators. We found that we could evaluate an impulse response (and thus a transfer function) of the system at an image point. Consider the two-lens system shown in the figure below, that is illuminated by “collimated” monochromatic radiation (i.e., plane waves). In words, the object $f[x, y]$ is illuminated with a converging wave from the first lens. The second lens produces an image of $f[x, y]$ at the plane labeled by $g[x, y]$. The pupil functions of the two lenses are respectively $p_1[x, y]$ and $p_2[x, y]$.
 - (a) Write down the mathematical equation for the amplitude located at the distance z_1 from the object.
 - (b) Evaluate the amplitude of the output image $g[x, y]$ in terms of $f[x, y]$ and the parameters of the system.
 - (c) Use the result of part (b) to find a condition on the distances z_1 , z_2 , and z_3 that must be satisfied for an image to be formed at the plane where $g[x, y]$ is observed.
 - (d) Locate the point in this system where the Fourier transform of the object exists and may be modified by introducing a multiplicative transfer function for the system.



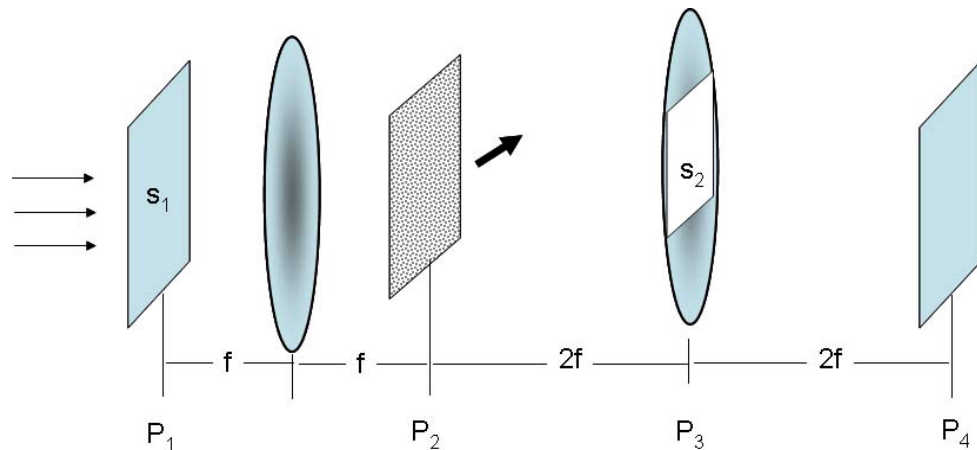
2. The MTF is defined for nonnegative sinusoidal functions in incoherent light that specifies how well the modulation is transferred as a function of spatial frequency. The analogous metric for square-wave functions is the *contrast transfer function* (CTF), which is measured for square waves with 50% duty cycle (50% “on” and 50% “off”). Derive an expression for the 1-D CTF $C[\xi]$ in terms of 1-D MTF $M[\xi]$ (hint: spectrum of square wave)

3. Consider the following optical imaging systems that operate in both monochromatic (coherent) and “quasimonochromatic” (incoherent) light centered at wavelength $\lambda_0 = 500$ nm. The systems operate at “equal conjugates” so that the object and image distances are identically $2\mathbf{f}_1$, where $\mathbf{f}_1 = +200$ mm. In other words, the complex amplitude transmittance of the lens is

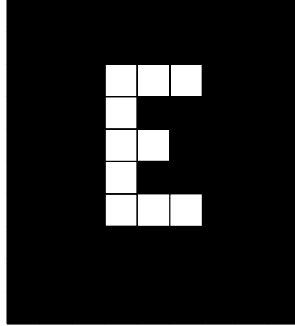
$$t[x, y] = p[x, y] \cdot \exp \left[-i\pi \left(\frac{x^2 + y^2}{\lambda_0 \mathbf{f}_1} \right) \right]$$

where $p[x, y]$ may be complex valued. Evaluate and plot graphical profiles of the impulse responses of the following optical imaging systems along the x - and y -axes AND of the transfer functions along the ξ - and η -axes.

- $p[x, y]$ is a square aperture with sides of length 50 mm. Find the spatial frequency where the MTF is 50% and find the “cutoff” frequency where the MTF first reaches 0 along both x - and y -axes.
 - $p[x, y]$ is a square aperture with sides of length 50 mm where the left half of the aperture (where $x < 0$) is covered with a sheet of glass of refractive index $n = 1.5$ and thickness $\tau_0 = 1000$ nm.
 - The pupil function consists of four square apertures with sides of 10 mm whose centers are arranged to form a square with sides 30 mm that is symmetrically placed about the optical axis.
 - The pupil function is identical to that in part (c) but one of the four apertures is overlaid with the same sheet of glass in part (c).
4. Consider the optical system shown in the figure. A transparency with a real non-negative *amplitude* transmittance (NOT transmittance of squared magnitude) $s_1[x, y]$ is placed in plane P_1 and illumination by a monochromatic, unit-intensity, normally incident plane wave. Lenses L_1 and L_2 are spherical with the same focal length \mathbf{f} . In plane P_2 , which is the focal plane of L_1 , a *moving* diffuser is placed. The effect of the moving diffuser can be considered to be the conversion of spatially coherent incident light into spatially incoherent transmitted light without changing the intensity distribution of light in plane P_3 , in contact with L_2 , is placed a second transparency, this one with *amplitude* transmittance $s_2[x, y]$. Find an expression for the intensity distribution on plane P_4 .



5. The bitonal transparency $f[x, y]$ of an upper-case letter “E” (transparent character on opaque background, as shown in the figure) is placed with its center at the origin of coordinates (there is no specific “reference” source). The transparency is illuminated by light with wavelength λ_0 and the transmitted light travels down the z -axis into the Fraunhofer diffraction region located at distance z_1 . The irradiance pattern is recorded on a photographic emulsion and processed to obtain a “linear” response and replaced at its original location. The transparency is reilluminated by light with the same wavelength from a point source located at the origin. The diffracted light propagates the distance z_2 to an observation plane.



Derive the amplitude and irradiance patterns that are reconstructed at the observation plane.

6. This problem demonstrates the utility of the stationary-phase approximation in imaging applications. Consider the 1-D modulated quadratic-phase function:

$$f[x] = \text{RECT} \left[\frac{x - 2.5}{5} \right] \exp [+i\pi x^2]$$

- Try to evaluate $f[x] * f[x]$ (I dare you!)
- Evaluate the stationary-phase approximation $\hat{F}[\xi]$ of the Fourier transform of $f[x]$ (easy)
- Use the result of part (b) to find an approximation for expression in part (a)
- Use the result of part (b) to find an approximation for $f[x] \star f[x] = f[x] * f^*[-x]$.
- Find an approximate expression for the Fourier transforms of the real and imaginary parts of $f[x]$.