SIMG-738-20063 Homework #5 Due 5/10/2007 (Th)

1. Goodman 6-2: The *line-spread function* of a two-dimensional imaging system is defined to be the response of that system to a one-dimensional Dirac delta function passing through the origin of the input plane.

   (a) In the case of a line excitation lying along the x-axis, show that the line-spread function $\ell[y]$ and the point spread function $p[x,y]$ are related by:

   $$\ell[y] = \int_{-\infty}^{+\infty} p[x,y]\ dx$$

   where ℓ and p are interpreted as amplitudes or intensities, depending on whether the system is coherent or incoherent, respectively.

   (b) Show that for a line source oriented along the x-axis, the 1-D Fourier transform of the line-spread function is equal to a slice through the 2D Fourier transform of the point-spread function along the ξ-axis. In other words, if $\mathcal{F}_1\{\ell[y]\} = L[\eta]$ and $\mathcal{F}_2\{p[x,y]\} = P[\xi,\eta]$, then $L[\eta] = P[0,\eta]$ (this is the so-called *central-slice theorem*).

   (c) Find the relationship between the lsf and the step response of the system, i.e., the response to a unit-step excitation oriented parallel to the x-axis.

   (d) (ADDITIONAL PART) This method may be adapted to evaluate the OTF of an imaging system with a nonsymmetric pupil, as from systems composed of multiple apertures. Describe how this may be accomplished.

2. Goodman 6-6:

   (a) Sketch the cross sections of the OTF of an incoherent imaging system whose pupil consists of two circular apertures of diameter $2w$ separated by $2d$. Label the various cutoff and center frequencies.

   (b) (ADDITIONAL PART) Repeat (a) for the case of an incoherent system where one of the two "subapertures" is misaligned in its cell to add a phase ϕ to the light through that aperture. For example, if the system is a reflector telescope, one of the two mirrors can "slip" in its cell by some amount Δz so that the optical path difference is $\phi = \frac{2\pi}{\lambda} \cdot \Delta z$. Evaluate for the cases $\phi = \frac{\pi}{4}$, $\frac{\pi}{2}$, and π radians.

3. Goodman 6-8: Consider the incoherent OTF for focusing error in Eq.(6-41):

$$\mathcal{H}[\xi,\eta] = TRI\left[\frac{\xi}{2f_o},\frac{\eta}{2f_o}\right]\ SINC\left[\frac{8W}{\lambda_0}\left(\frac{\xi}{2f_o}\right)\left(1-\frac{|\xi|}{2f_o}\right)\right] \times SINC\left[\frac{8W}{\lambda_0}\left(\frac{\eta}{2f_o}\right)\left(1-\frac{|\eta|}{2f_o}\right)\right] \quad (6\text{-}41)$$

   as predicted for a system having a square pupil and focusing error. It is hypothesized that the psf of this system is the convolution of the diffraction-limited psf with the psf predicted by geometrical optics. Examine the validity of this claim.

4. Goodman 6-9: A metric of considerable utility in determining the "seriousness" of aberrations of an optical system is the *Strehl definition* $\mathcal{D}$, which is defined as the ratio of the light intensity at the maximum of the psf of the system with aberrations to that same maximum for an aberration-free system. (Both maxima are assumed to exist on the optical axis). Prove that $\mathcal{D}$ is equal to the normalized volume under the OTF of the aberrated system; that is, prove

$$\mathcal{D} = \frac{\iint_{-\infty}^{+\infty} \mathcal{H}\left[\xi, \eta\right]_{\text{with aberrations}} \; d\xi \; d\theta}{\iint_{-\infty}^{+\infty} \mathcal{H}\left[\xi, \eta\right]_{\text{without aberrations}} \; d\xi \; d\theta}$$

5. Gooodman 6-13: The *F-number* of a lens with a circular aperture is defined as the ratio of the focal length to the lens diameter. Show that thwn the object distance is infinite, the cutoff frequency for a coherent imaging system using this lens is given by

$$f_o = \frac{1}{2 \; \lambda \; F/\#}$$