

0. Read Goodman §9 *Holography* sections 9.1 - 9.5 and skim 9.9 - 9.12
1. Goodman 9-1: A hologram is recorded using a spherical reference wave that is diverging from the point  $[x_r, y_r, z_r]$  and the images from that hologram are played back with a reconstruction beam that is diverging from the point  $[x_p, y_p, z_p]$ . The wavelength used for both recording and reconstruction is  $\lambda_1$ . The hologram is taken to be circular with diameter  $D$ . It is claimed that the image of an arbitrary three-dimensional object obtained by this method is entirely equivalent to that obtained by a lens of the same diameter and the same distance from the object and a prism, where again the wavelength is  $\lambda_1$ . What are the two possible focal lengths for the lens that produce equivalence?
  2. Goodman 9-3: A hologram is recorded and its images reconstructed with the same wavelength  $\lambda$ . Assuming that  $z_o < 0$ , show that when  $z_p = z_r$ , there results a virtual image with unity transverse magnification, whereas with  $z_p = -z_r$ , there results a real image with unity transverse magnification. What is the transverse magnification of the twin image in each case?
  3. Goodman 9-6: It is proposed to record an X-ray hologram using coherent radiation of wavelength  $\lambda = 0.1 \text{ nm}$  and to reconstruct the images optically using light with  $\lambda = 600 \text{ nm}$ . The object is a square transparency with a pattern of absorption at the X-ray wavelength. The lensless Fourier transform recording geometry is chosen (Goodman Figure 9.14 p. 322). The width of the object is  $100 \mu\text{m}$  and the minimum distance between the object and the reference is to be  $200 \mu\text{m}$  to assure that the twin images will be separated from “on-axis” interference. The X-ray film is placed  $20 \text{ mm}$  from the object.
    - (a) What is the maximum spatial frequency (cycles per mm) in the interference pattern falling on the film?
    - (b) Assume that the film has sufficient resolution to record all of the incident intensity variations. It is proposed to reconstruct the images in the usual manner, i.e., by looking in the rear focal plane of a Fourier transforming lens. What will this experiment fail?

4. Goodman 9-15: A certain emulsion has a nonlinear curve of amplitude transmittance  $t_A$  vs. exposure  $E$ :

$$t_A = t_b + \beta E_1^3$$

where  $E_1$  is the range of exposure variation about the reference exposure and  $t_b$  is the uniform “bias” transmittance established by the constant reference exposure.

- (a) Assuming a planar reference wave  $A \exp[-2\pi i \alpha x]$  and an object wave:

$$a[x, y] \exp[-i\phi[x, y]]$$

at the film, find an expression for that portion of the transmitted field that generates twin first-order images.

- (b) To what does this expression reduce if  $A \gg |a|$ ?
- (c) How do the amplitude and phase modulations obtained in the previous parts of the problem compare with the ideal amplitude and phase modulations present when the film as a linear curve of  $t_A$  vs.  $E$ ?