

1. The pupil function of the lens in an imaging system has the form:

$$p[x, y] = CYL \left(\frac{\sqrt{x^2 + y^2}}{d_0} \right) * (\delta[x + x_0, y] + \delta[x - x_0, y])$$

where $d_0 \ll x_0, y_0$. Light through each small opening interferes to create a fringe pattern at the image plane.

- (a) Determine the spatial frequency of the fringe in terms of the center-to-center spacing between the openings, the wavelength λ_0 , and the image distance z_2 .
 - (b) Specify the pattern envelope due to the finite aperture diameter.
2. For the pupil function in the previous problem, sketch the cross sections of the incoherent transfer function. Label the various cutoff frequencies and center frequencies on the graphs.
3. Consider a pinhole camera where the pupil function (circular pinhole of diameter d_0) is small compared to the object distance z_1 , which may be approximated as infinite. Assume that the object is illuminated by quasimonochromatic light with mean wavelength λ_μ .
- (a) Assuming that the pinhole is small, but sufficiently large so that ray optics is valid, find the optical transfer function of the camera. Determine the spatial frequency of the first zero of the OTF (which we may call the “cutoff frequency”).
 - (b) Now assume that the pinhole is so small that Fraunhofer diffraction governs the shape of the psf; calculate the spatial frequency of the first zero of the OTF.
 - (c) Use the results of parts (a) and (b) to estimate an “optimum” size of the pinhole in terms of the various parameters, i.e., the diameter that produces the largest cutoff frequency (of the first zero).
4. A transparent object has transmittance $t[x, y] = \left(RECT \left[\frac{x}{b_0} \right] * \frac{1}{d_0} COMB \left[\frac{x}{d_0} \right] \right) \cdot 1[y]$, where $d_0 = \frac{1}{100}$ mm $> b_0$. The object is imaged by a lens with a circular pupil and focal length $f = 100$ mm. The illumination is monochromatic with $\lambda_0 = 1 \mu\text{m}$.
- (a) Determine the minimum lens diameter that will produce an image with any variation of intensity due to the object.
 - (b) Determine the minimum diameter that produces an image with the same feature if the illumination is quasimonochromatic with $\lambda_\mu = 1 \mu\text{m}$.
5. The $f/\#$ of a lens with a circular aperture is the ratio of the focal length to the diameter. Under coherent illumination with λ_0 and for an infinite object distance, find an expression for the cutoff frequency.