1. Goodman 4-12: Consider a thin periodic grating whose amplitude transmittance can be represented by the complex Fourier series (my notation)

\[
t_A[x] = \sum_{k=-\infty}^{+\infty} c_k \exp \left[+2\pi i \frac{x}{L}\right]
\]

where \( L \) is the period of the grating and the coefficients \( c_k \) are:

\[
c_k = \frac{1}{L} \int_{-\frac{L}{2}}^{+\frac{L}{2}} t_A[x] \cdot \exp \left[+2\pi i \frac{k x}{L}\right] dx
\]

Neglect the aperture that bounds the grating, since it will not affect the quantities of interest here.

(a) Show that the diffraction efficiency into the \( k^{th} \) order of the grating is simply

\[
\eta_k = |c_k|^2
\]

(b) Calculate the diffraction efficiency into the first diffraction order for a grating with amplitude transmittance given by:

\[
t_A[x] = \left| \cos \left(\frac{\pi x}{L}\right) \right|
\]

2. Goodman 4-13: The amplitude transmittance function of a thin square-wave absorption grating is shown in the figure. Find the following properties of this grating:

(a) The fraction of incident light that is absorbed by the grating;
(b) The fraction of incident light that is transmitted by the grating;
(c) The fraction of incident light that is transmitted into a single first order.

3. Goodman 4-17: Find the intensity distribution on the aperture axis in the Fresnel diffraction patterns of apertures with the following transmittance functions (assume normally incident, unit-amplitude plane-wave illumination):
(a) \[
t_A [x, y] = CYL \left( \sqrt{x^2 + y^2} \right)
\]

(b) \[
t_A [x, y] = \begin{cases} 
1 & a \leq \sqrt{x^2 + y^2} \leq b \\
0 & \text{otherwise}
\end{cases}
\]

where \(a < 1\), \(b < 1\), and \(a < b\).

4. Goodman 5-7: A normally incident, unit-amplitude, monochromatic plane wave illuminates a converging lens of diameter 50 mm and focal length \(f = 2 \text{ m}\). An object with amplitude transmittance (Easton notation)

\[
t_A [x, y] = \frac{1}{2} (1 + \cos (2\pi \xi_0 x)) \text{ RECT} \left[ \frac{x}{L}, \frac{y}{L} \right]
\]

Assuming \(L = 10 \text{ mm}, \lambda = 633 \text{ nm}\) and \(\xi_0 = 10^{\text{cycles mm}}\), sketch the intensity distribution across the \(x\) axis of the focal plane, labeling the numerical values of the distances between diffracted components and the width (between first zeros) of the individual components.

5. Goodman 6-3: An incoherent imaging system has a square pupil of width \(2w\). A square stop of width \(w\) may be placed in the center of the pupil.

(a) Sketch cross sections of the optical transfer function both without and with the stop present.

(b) Sketch the limiting form of the optical transfer function as the size of the stop approaches the size of the full pupil.

6. Goodman 6-7: Consider a pinhole camera with an object illuminated by incoherent and nearly monochromatic light, that the distance \(z_1\) from the object to the pinhole is so large that it can be treated as infinite, and the pinhole is circular with diameter \(2w\).

(a) Under the assumption that the pinhole is sufficiently large to allow a purely geometrical-optics estimation of the point-source function, find the optical transfer function of this camera. If we define the “cutoff frequency” of the camera to be the frequency where the first zero of the OTF occurs, what is the cutoff frequency under the above geometrical-optics approximation? (Hint: first find the intensity point-spread function, then evaluate the Fourier transform. Remember the second approximation above.

(b) Again, calculate the cutoff frequency, but this time assuming that the pinhole is so small that Fraunhofer diffraction by the pinhole governs the shape of the point-spread function.

(c) Considering the two expressions for the cutoff frequency that you have found, can you estimate the “optimum” size of the pinhole in terms of the various parameters of the system? Optimum in this case means the size that produces the highest possible cutoff frequency.