1051-738-20083 Homework #2 Due 4/9/2009 (Th)

1. An array of one-dimensional input functions can be represented by $f_o[x, y_n]$, where y_n are N fixed coordinates in the input plane. Your task is to perform 1-D Fourier transforms on all N 1-D functions along the x-direction to produce the array of transforms:

$$F_o[\xi; y_n] = \int_{-\infty}^{+\infty} f_o[x, y_n] \ \exp[-2\pi i x \xi] \ dx$$

After neglecting the finite extent of the apertures of the object and the lens, use the Fourier transforming and imaging properties of lenses derived in this chapter to show how to compute the squared magnitude $|F_o[\xi; y_n]|^2$ with the lenses available in two cases:

   (a) two cylindrical lenses with different focal lengths

   (b) a cylindrical and a spherical lens with the same focal length.

2. A normally incident unit-amplitude monochromatic plane wave with $\lambda_0 = 633\,\text{nm}$ illuminates a converging lens with diameter $d_0 = 50\,\text{mm}$ and $\mathbf{f} = 2\,\text{m}$. An object with trnasmittance

$$t[x, y] = \frac{1}{2}\left(1 + \cos\left[2\pi \cdot 10\,\text{mm}^{-1} \cdot x\right]\right) RECT\left[\frac{x}{10\,\text{mm}}, \frac{y}{10\,\text{mm}}\right]$$

is placed one meter *behind* the lens (i.e., on the "output" side). Sketch the inrradiance across the x-axis in the focal plane; label the numerical values of the distance between the diffracted components and the width of the individual components between the first zeros.

3. A monochromatic point source is placed at a fixed distance z_1 to the left of a positive lens with focal length $\mathbf{f}$, where $z_1 > \mathbf{f}$. A transparent object with transmittance $t[x, y]$ is placed at a variable distance ℓ to the left of the lens. The Fourier transform and the image of the object appear to the right of the lens.

   (a) Find ℓ in terms of z_1 and $\mathbf{f}$, that assures that the *Fourier plane* and the *object* are equidistant from the lens.

   (b) When the object is located at the distance ℓ found in part (a), determine the location of the image and its transverse magnification.

4. Two lenses L_1 with focal length $\mathbf{f}_1 = -|\mathbf{f}|$ and L_2 with $\mathbf{f}_2 = +|\mathbf{f}|$ are separated by the distance $\mathbf{f}_2$. The object is located at the distance $2 \cdot |\mathbf{f}|$ to the left of L_1. Find the distances to the Fourier plane and the image plane and sketch the system.

5. A converging spherical wave illuminates a transparency $t[x, y]$ and converges to a point at the distance $\mathbf{f}$ to its right. A lens with focal length $+|\mathbf{f}|$ is placed at the distance $2 \cdot \mathbf{f}$ to the right of the focal point. Find the locations of all Fourier and image planes on both sides of the lens.