0.1 OPTICAL IMAGING

The complete expression for the complex amplitude in the image plane includes the leading quadratic-phase factor:

$$\mathcal{O}\left\{f\left[\frac{x}{M},\frac{y}{M}\right]\right\} = \frac{M}{(\lambda z_2)^2} e^{+2\pi i \frac{z_1+n\ell_0+z_2}{\lambda}} \; e^{+\frac{i\pi}{\lambda z_2}\left[\left(x^2+y^2\right)\left(1-\frac{1}{M}\right)\right]}$$
$$\times \iint_{-\infty}^{+\infty} f\left[\frac{x_0}{M},\frac{y_0}{M}\right] \; P\left[\frac{x-Mx_0}{\lambda z_2},\frac{y-My_0}{\lambda z_2}\right] \; dx_0 \; dy_0$$

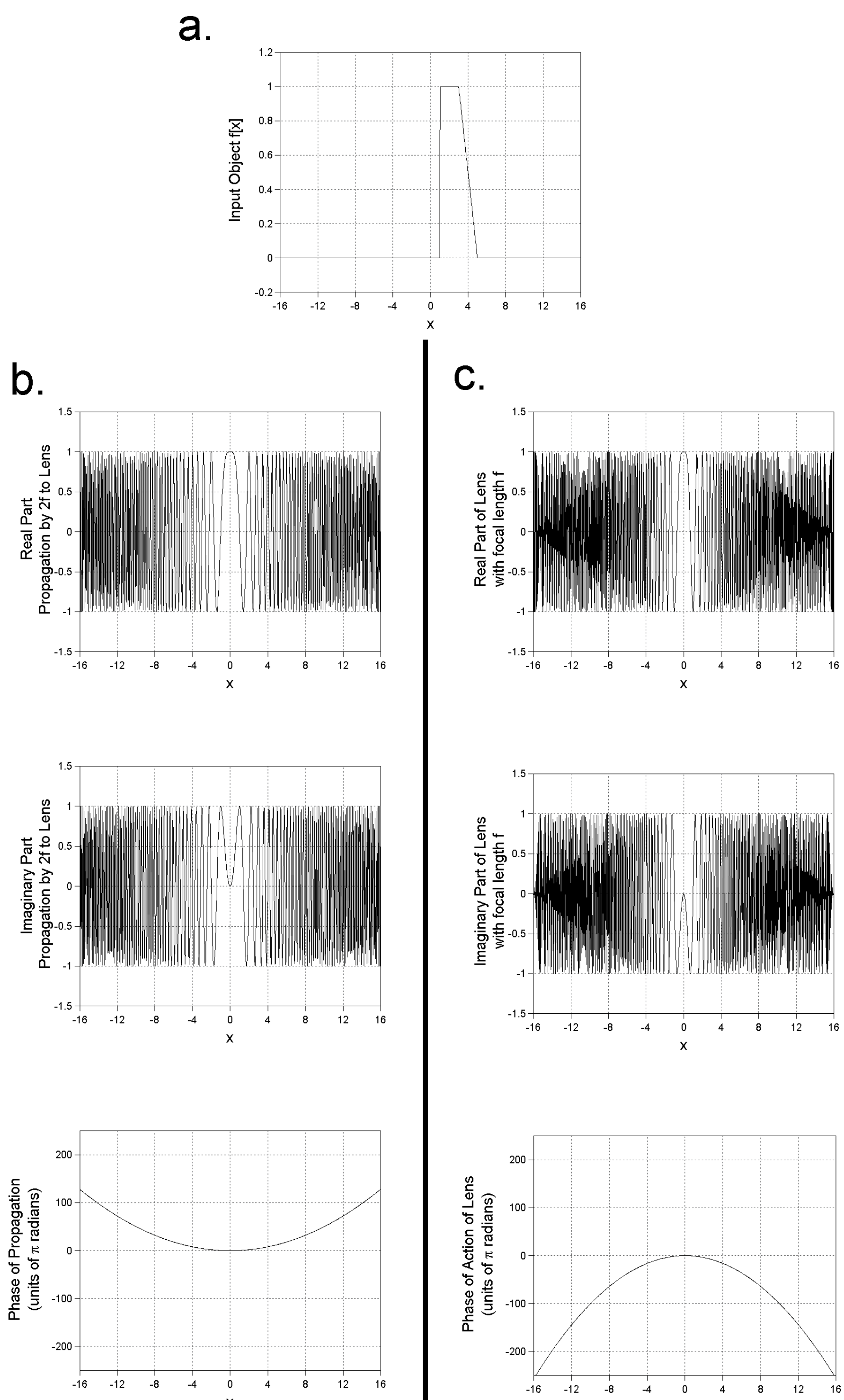
It is occasionally convenient to rewrite the leading quadratic-phase factor in terms of the "invariant" focal length and the transverse magnfication:

$$\exp\left[+\frac{i\pi}{\lambda z_2}\left[\left(x^2+y^2\right)\left(1-\frac{1}{M}\right)\right]\right] = \exp\left[+\frac{i\pi}{\lambda\left[(1-M)\,f\right]}\left[\left(x^2+y^2\right)\left(\frac{M-1}{M}\right)\right]\right]$$
$$= \exp\left[-i\pi\frac{x^2+y^2}{\alpha^2}\right] \textit{ where } \alpha = \sqrt{M\lambda f}$$

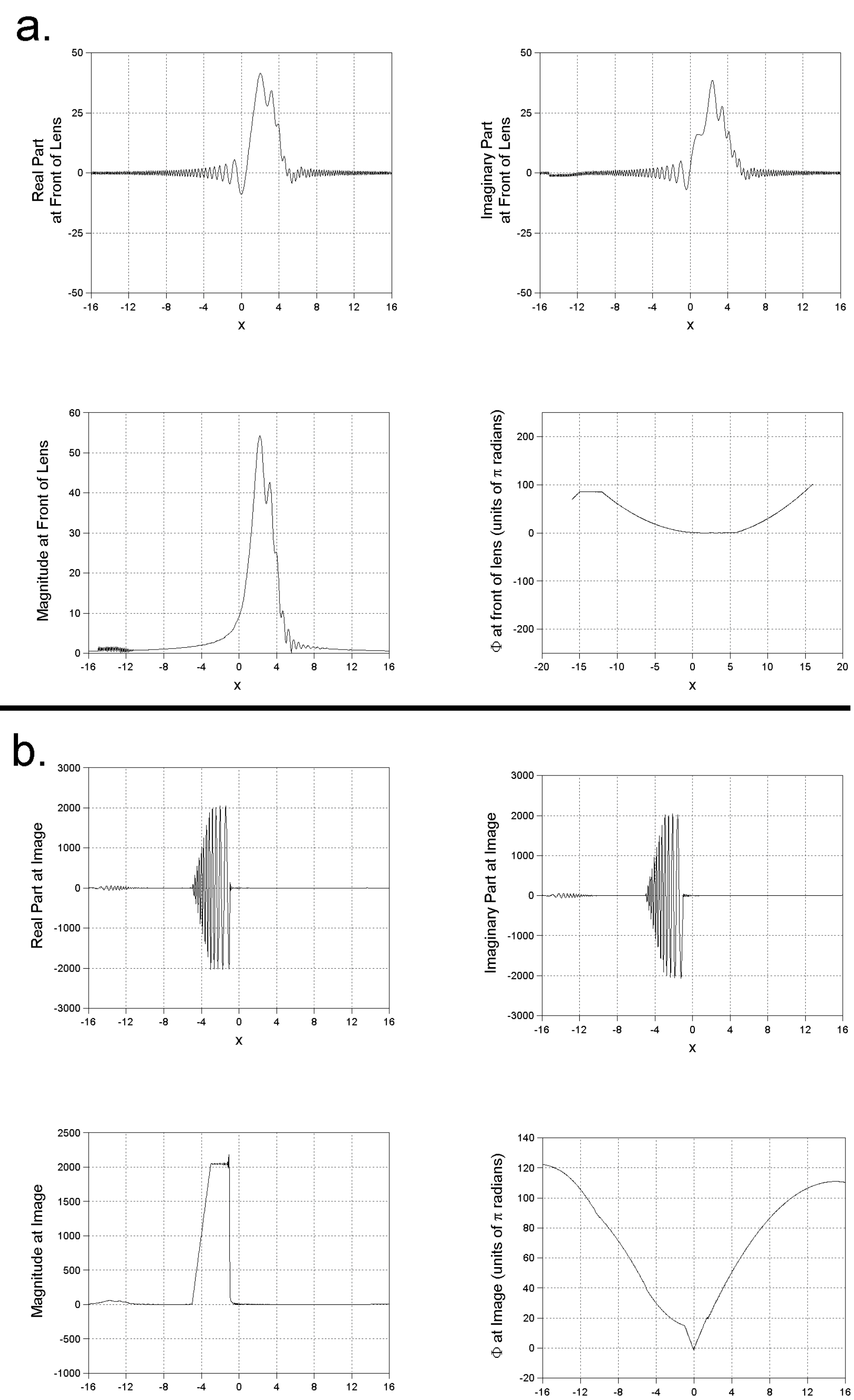
Evaluate the chirp rate α in some imaging cases. First, consider imaging at equal conjugates, so that $z_1 = z_2 = 2f$ and the transverse magnification is $M = -1$:

$$\exp\left[+\frac{i\pi}{\lambda z_2}\left(x^2+y^2\right)\left(1-\frac{1}{M}\right)\right] = \exp\left[+i\pi\frac{x^2+y^2}{\lambda f}\right]$$

The "residual" quadratic-phase function has a positive chirp rate (diverging wave).



(a) 1-D test object $f[x]$; (b) impulse response in Fresnel approximation of propagation with $z = 2f$ as real and imaginary parts and as phase (magnitude is unity); (c) lens function for focal length f assuming “infinite” support as real and imaginary parts and as phase.

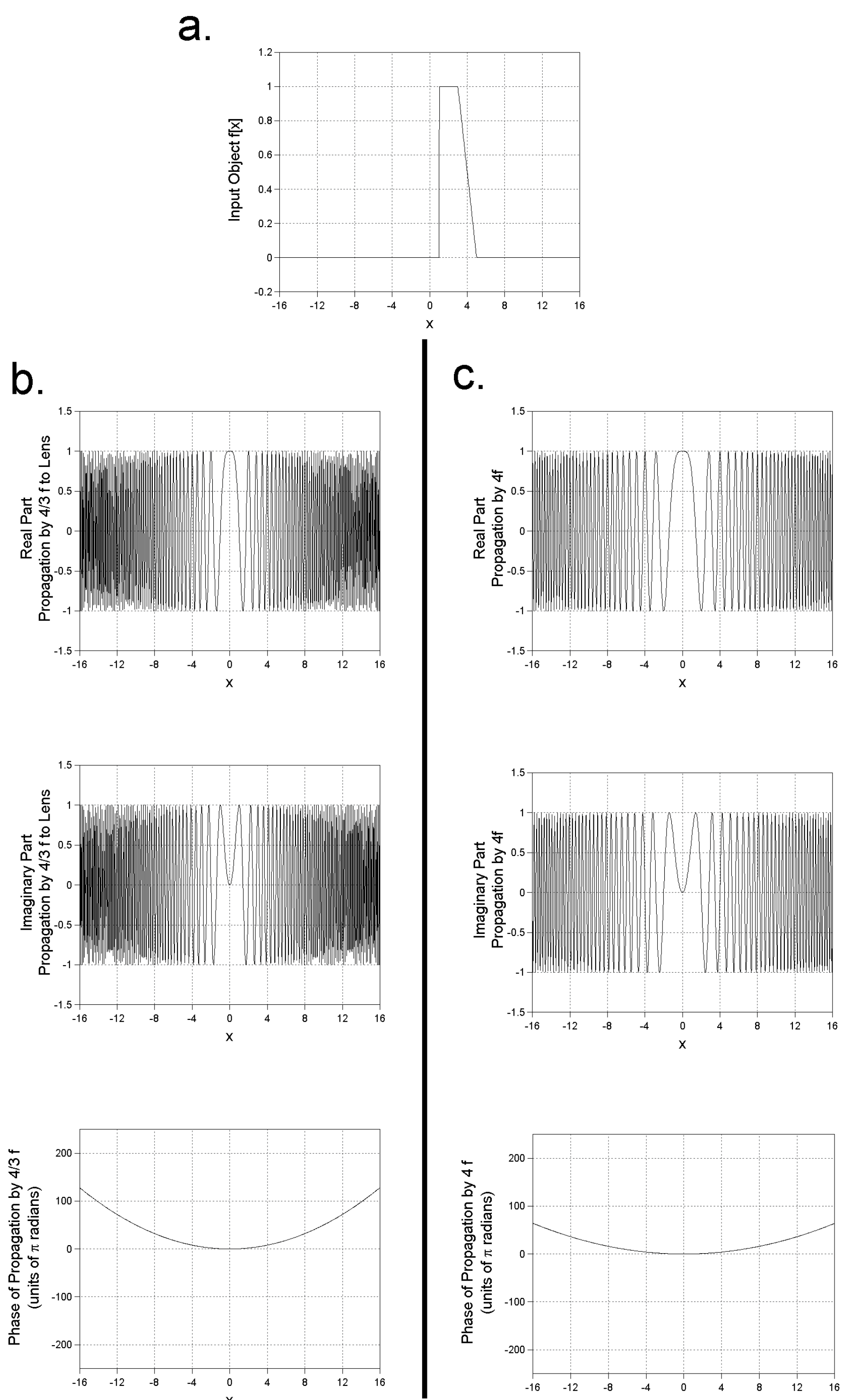


Imaging of test object at equal conjugates: (a) amplitude after propagation by distance $z_1 = 2f$ to lens of focal length f as real part, imaginary part, magnitude, and phase; (b) amplitude distribution at image plane as real part, imaginary part, magnitude, and phase, showing residual diverging quadratic-phase factor.

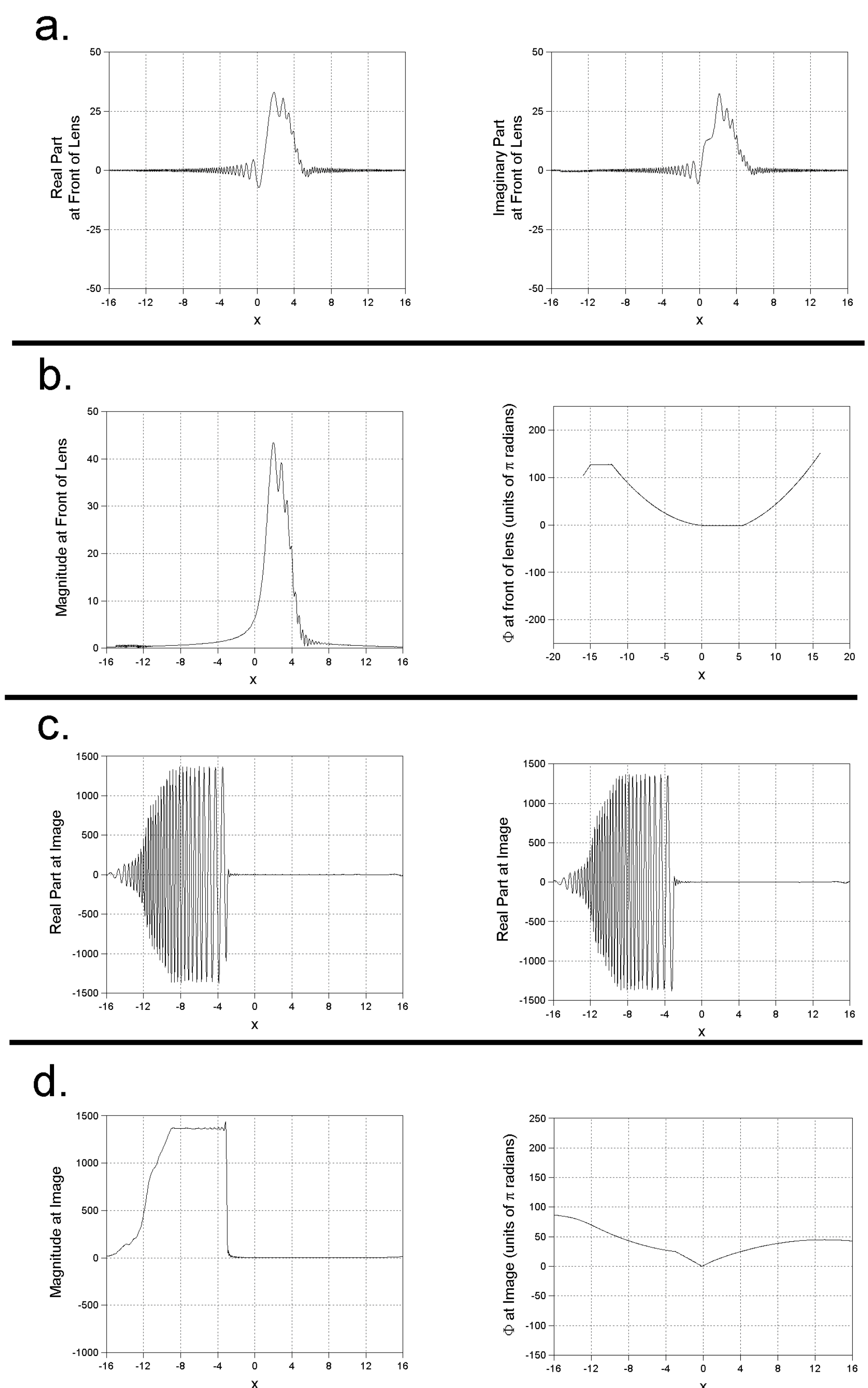
Now consider a system that "minifies" the image. If the object distance is $z_1 = \frac{4}{3}f$, the corresponding image distance is $z_2 = 4f$, $M = --3$, and $\alpha = \sqrt{-3\lambda f}$

$$\exp\left[+\frac{i\pi}{\lambda z_2}\left(x^2+y^2\right)\left(1+\frac{1}{3}\right)\right] = \exp\left[+i\pi\frac{x^2+y^2}{3\lambda f}\right]$$

Because the magnification is negative for all real images, the residual chirp rate is a diverging spherical wave.



(a) 1-D test object $f[x]$; (b) impulse response of propagation by distance $z_1 = \frac{4}{3}f$ in Fresnel approximation as real part, imaginary part, and phase (magnitude is unity); (c) impulse response of propagation by distance $z_1 = 4f$ in Fresnel approximation as real part, imaginary part, and phase (magnitude is unity).



Imaging with distances $z_1 = \frac{4}{3}f$ and $z_2 = 4f$, so that $M = -3$; (a) amplitude at front of lens as real part, imaginary part, magnitude and phase; (b) amplitude at image plane as real part, imaginary part, magnitude and phase, showing magnification and residual quadratic phase.