1051-733-20072 **Homework #2 Solutions**

1.

$$h[x,y;z_1,\lambda_0]=\int\int_{-\infty}^{\infty}\left(\frac{1}{i\lambda_0 z_1}e^{i\frac{\pi}{\lambda_0 z_1}(x^2+y^2)}\right)dxdy$$

$$=\frac{1}{i\lambda_0 z_1}\int_{-\infty}^{\infty}\left(e^{i\frac{\pi}{\lambda_0 z_1}(x^2)}\right)dx\int_{-\infty}^{\infty}\left(e^{i\frac{\pi}{\lambda_0 z_1}(y^2)}\right)dy$$

We will use the central ordinate theorem to evaluate the integrals.

$$=\frac{1}{i\lambda_0 z_1}F(\xi=0)\left\{e^{i\frac{\pi}{\lambda_0 z_1}(x^2)}\right\}F(\xi=0)\left\{e^{i\frac{\pi}{\lambda_0 z_1}(y^2)}\right\}$$

Making use of the chirp FT pair:

$$e^{\pm i\pi\left(\frac{x}{\alpha}\right)^2}\Leftrightarrow|\alpha|e^{\pm i\frac{\pi}{4}}e^{\mp i\pi(\alpha\xi)^2}$$

Gives us:

$$F\left\{e^{i\frac{\pi}{\lambda_0 z_1}(x^2)}\right\}=\sqrt{\lambda_0 z_1}\,e^{i\frac{\pi}{4}}e^{-i\pi\left(\sqrt{\lambda_0 z_1}\,\xi\right)^2}$$

Substituting back into our expression for h:

$$=\frac{1}{i\lambda_0 z_1}\sqrt{\lambda_0 z_1}\,e^{i\frac{\pi}{4}}e^{-i\pi\left(\sqrt{\lambda_0 z_1}\,\xi\right)^2}\sqrt{\lambda_0 z_1}\,e^{i\frac{\pi}{4}}e^{-i\pi\left(\sqrt{\lambda_0 z_1}\,\eta\right)^2}$$

$$=\frac{1}{i\lambda_0 z_1}\sqrt{\lambda_0 z_1}\,e^{i\frac{\pi}{4}}e^{-i\pi\left(\sqrt{\lambda_0 z_1}\,(0)\right)^2}\sqrt{\lambda_0 z_1}\,e^{i\frac{\pi}{4}}e^{-i\pi\left(\sqrt{\lambda_0 z_1}\,(0)\right)^2}$$

$$=\frac{1}{i\lambda_0 z_1}\lambda_0 z_1 e^{i\frac{\pi}{2}}=\frac{i}{i}=1$$

2.

The propagation is given by convolution with an impulse response.

$f[x, y] * h[x, y; z]$

Where the impulse response for Fresnel diffraction is given by:

$$h[x, y; z] = \frac{1}{i\lambda_0 z} e^{2\pi i\left(\frac{z}{\lambda_0} - \nu t\right)} e^{\frac{i\pi}{\lambda_0 z}\left(x^2+y^2\right)}$$

We want to prove $h[x, y; z_1] * h[x, y; z_2] = h[x, y; z_1 + z_2]$.

$$h[x, y; z_1] * h[x, y; z_2] = \frac{1}{i\lambda_0 z_1} e^{2\pi i\left(\frac{z_1}{\lambda_0} - \nu t_1\right)} e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)} * \frac{1}{i\lambda_0 z_2} e^{2\pi i\left(\frac{z_2}{\lambda_0} - \nu t_2\right)} e^{\frac{i\pi}{\lambda_0 z_2}\left(x^2+y^2\right)}$$

$$= \frac{1}{i\lambda_0 z_1} \frac{1}{i\lambda_0 z_2} e^{2\pi i\left(\frac{z_1}{\lambda_0} - \nu t_1\right)} e^{2\pi i\left(\frac{z_2}{\lambda_0} - \nu t_2\right)} \left[e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)} * e^{\frac{i\pi}{\lambda_0 z_2}\left(x^2+y^2\right)} \right]$$

$$= \frac{-1}{\lambda_0{}^2 z_1 z_2} e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)} \left[\left(e^{\frac{i\pi}{\lambda_0 z_1}x^2} * e^{\frac{i\pi}{\lambda_0 z_2}x^2} \right) \left(e^{\frac{i\pi}{\lambda_0 z_1}y^2} * e^{\frac{i\pi}{\lambda_0 z_2}y^2} \right) \right]$$

Using the FT of a chirp:

$$= \frac{-1}{\lambda_0{}^2 z_1 z_2} e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)} \left[\left(e^{\frac{i\pi}{\lambda_0 z_1}x^2} * e^{\frac{i\pi}{\lambda_0 z_2}x^2} \right) \left(e^{\frac{i\pi}{\lambda_0 z_1}y^2} * e^{\frac{i\pi}{\lambda_0 z_2}y^2} \right) \right]$$

$$= \frac{-1}{\lambda_0{}^2 z_1 z_2} e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)}$$

$$\cdot \left[F^{-1}\left\{ \sqrt{\lambda_0 z_1} e^{i\frac{\pi}{4}} e^{-i\pi\left(\sqrt{\lambda_0 z_1}\xi\right)^2} \sqrt{\lambda_0 z_2} e^{i\frac{\pi}{4}} e^{-i\pi\left(\sqrt{\lambda_0 z_2}\xi\right)^2} \right\} F^{-1}\left\{ \sqrt{\lambda_0 z_1} e^{i\frac{\pi}{4}} e^{-i\pi\left(\sqrt{\lambda_0 z_1}\eta\right)^2} \sqrt{\lambda_0 z_2} e^{i\frac{\pi}{4}} e^{-i\pi\left(\sqrt{\lambda_0 z_2}\eta\right)^2} \right\} \right]$$

$$= \frac{-1}{\lambda_0{}^2 z_1 z_2} e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)} \sqrt{\lambda_0 z_1} e^{i\frac{\pi}{4}} \sqrt{\lambda_0 z_2} e^{i\frac{\pi}{4}} \sqrt{\lambda_0 z_1} e^{i\frac{\pi}{4}} \sqrt{\lambda_0 z_2} e^{i\frac{\pi}{4}}$$

$$\cdot \left[F^{-1}\left\{ e^{-i\pi\left(\sqrt{\lambda_0 (z_1+z_2)}\xi\right)^2} \right\} F^{-1}\left\{ e^{-i\pi\left(\sqrt{\lambda_0 (z_1+z_2)}\eta\right)^2} \right\} \right]$$

$$= e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)} \left[F^{-1}\left\{ e^{-i\pi\left(\sqrt{\lambda_0 (z_1+z_2)}\xi\right)^2} \right\} F^{-1}\left\{ e^{-i\pi\left(\sqrt{\lambda_0 (z_1+z_2)}\eta\right)^2} \right\} \right]$$

Once more using the FT of a chirp:

$$= e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)} \frac{1}{\sqrt{\lambda_0 (z_1+z_2)}} e^{-i\frac{\pi}{4}} e^{i\pi\left(\frac{x}{\sqrt{\lambda_0 (z_1+z_2)}}\right)^2} \frac{1}{\sqrt{\lambda_0 (z_1+z_2)}} e^{-i\frac{\pi}{4}} e^{i\pi\left(\frac{y}{\sqrt{\lambda_0 (z_1+z_2)}}\right)^2}$$

$$= \frac{1}{i\lambda_0 (z_1+z_2)} e^{2\pi i\left(\frac{z_1+z_2}{\lambda_0} - \nu (t_1+t_2)\right)} e^{i\pi\frac{x^2+y^2}{\lambda_0 (z_1+z_2)}}$$

The result is identical to our Fresnel impulse response at $t = t_1 + t_2$ and $z = z_1 + z_2$.

3.

(a)

The field due to a spherical wave from a source at (x_0, y_0, z_0) is given by:

$S[x,y,z,t] = \frac{E_0}{|\underline{r}|}\cos[\underline{k_0} \bullet \underline{r} - w_0 t]$ where $|\underline{k_0}| = \frac{2\pi}{\lambda}$ and

$|\underline{r}| = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$, the distance from the source.

The phase is given by $\Phi[x,y,z,t] = \underline{k_0} \bullet \underline{r} - w_0 t$ and its phase distribution by

$$|\Phi[x,y,z,t]| = |\underline{k_0}||\underline{r}| - w_0 t = \frac{2\pi}{\lambda}\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} - w_0 t.$$

Our source is located at $(0,0,-z_1)$ with wavelength λ_0 and we are interested in the distribution at $z=0$. Making these substitutions gives us:

$$|\Phi[x,y,z=0,t]| = \frac{2\pi}{\lambda_0}\sqrt{x^2+y^2+z_1^{\ 2}} - w_0 t = \frac{2\pi}{\lambda_0} z_1\sqrt{1+\frac{x^2+y^2}{z_1^{\ 2}}} - w_0 t$$

(b)

We can write $|\underline{r}|$ as a power series using the binomial theorem,

$$(1+\alpha)^n = \sum_{r=0}^{\infty}\frac{n!}{(n-r)!\,r!}\alpha^r.$$

$$|\underline{r}| = z_1\sqrt{1+\frac{x^2+y^2}{z_1^{\ 2}}} = z_1\left(1+\frac{x^2+y^2}{2z_1^{\ 2}} - \frac{(x^2+y^2)^2}{8z_1^{\ 2}} + \dots\right)$$

If z_1 is sufficiently large the terms of order 2 and larger can be assumed zero.

$$|\underline{r}| \approx z_1\left(1+\frac{x^2+y^2}{2z_1^{\ 2}}\right)$$

Using value in the phase distribution gives us:

$$|\Phi[x,y,z=0,t]| = \frac{2\pi}{\lambda_0} z_1\left(1+\frac{x^2+y^2}{2z_1^{\ 2}}\right) - w_0 t$$

(c)

The difference in phase can be found by subtraction our answers from part a and part b.

$$\Delta\Phi = |\Phi|_{spherical} - |\Phi|_{parabolic}$$

$$= \left(\frac{2\pi}{\lambda_0} z_1 \sqrt{1+\frac{x^2+y^2}{z_1^{\ 2}}} - w_0 t\right) - \left(\frac{2\pi}{\lambda_0} z_1 \left(1+\frac{x^2+y^2}{2z_1^{\ 2}}\right) - w_0 t\right)$$

$$= \frac{2\pi}{\lambda_0} z_1 \left(\sqrt{1+\frac{x^2+y^2}{z_1^{\ 2}}} - \left(1+\frac{x^2+y^2}{2z_1^{\ 2}}\right)\right)$$

This difference is equivalent to the terms we assumed were zero when making the paraxial approximation.

$$\Delta\Phi = -\frac{\left(x^2+y^2\right)^2}{8z_1^{\ 2}} + \ldots = \sum_{r=2}^{\infty} \frac{(1/2)!}{(1/2-r)!\,r!}\left(\frac{x^2+y^2}{z_1^{\ 2}}\right)^r$$

Since the leading term is negative the difference is negative and the spherical wavefront lags behind the parabolic one. This makes sense as the spherical wavefront has a greater curvature.

4.

To answer this question we need to consider the error of the converging spherical wave. Following a procedure similar to the last problem the phase distribution is given by

$$\left|\Phi[x,y,z,t]\right| = \frac{2\pi}{\lambda_0}(z-z_0)\sqrt{1+\frac{x^2+y^2}{(z-z_0)^2}} - w_0 t \approx \frac{2\pi}{\lambda_0}(z-z_0)\left(1+\frac{x^2+y^2}{2(z-z_0)^2}\right) - w_0 t$$

With an error of:

$$\Delta\Phi = |\Phi|_{spherical} - |\Phi|_{parabolic}$$

$$= \left(\frac{2\pi}{\lambda_0}(z-z_0)\sqrt{1+\frac{x^2+y^2}{(z-z_0)^2}} - w_0 t\right) - \left(\frac{2\pi}{\lambda_0}(z-z_0)\left(1+\frac{x^2+y^2}{2(z-z_0)^2}\right) - w_0 t\right)$$

$$= \frac{2\pi}{\lambda_0}(z-z_0)\left(\sqrt{1+\frac{x^2+y^2}{(z-z_0)^2}} - \left(1+\frac{x^2+y^2}{2(z-z_0)^2}\right)\right)$$

This time however $(z-z_0)$ is negative giving us a positive difference and a spherical wavefront that leads the parabolic one. The errors due to the paraxial approximation of the illuminating converging spherical wave and the error due to using the quadratic-phase approximation in the Fresnel diffraction equation partial cancel each other.

Complete cancelation will occur when the errors from the two wavefronts sum to zero. This occurs when $z - z_0 = z$ or $z = \frac{z_0}{2}$.

5.

(a)

As with earlier problems the diffraction pattern is found by convolution with the impulse response (problem 2).

$$f[x,y;0]*h[x,y;z_1]$$

$$=\left(\delta[x+x_0]+\delta[x-x_0]\right)\cdot 1[y]*\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)}$$

$$=\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\cdot\left(\left(\delta[x+x_0]+\delta[x-x_0]\right)*e^{\frac{i\pi}{\lambda_0 z_1}x^2}\right)\cdot\left(1[y]*e^{\frac{i\pi}{\lambda_0 z_1}y^2}\right)$$

We proceed by simplifying the equation with respect to x by convolving with the deltas, and with respect to y by using the FT of a constant and a chirp.

$$=\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(e^{\frac{i\pi}{\lambda_0 z_1}(x-x_0)^2}+e^{\frac{i\pi}{\lambda_0 z_1}(x+x_0)^2}\right)F^{-1}\left\{\delta(\eta)\sqrt{\lambda_0 z_1}e^{\frac{i\pi}{4}}e^{-i\pi\left(\frac{\eta}{\sqrt{\lambda_0 z_1}}\right)^2}\right\}$$

Rearranging the x terms and performing multiplication with a delta on the y terms gives us:

$$=\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+x_0{}^2\right)}\left(e^{\frac{i\pi}{\lambda_0 z_1}2xx_0}+e^{-\frac{i\pi}{\lambda_0 z_1}2xx_0}\right)F^{-1}\left\{\sqrt{\lambda_0 z_1}e^{\frac{i\pi}{4}}\right\}$$

Making a substitution using $2\cos x=e^{ix}+e^{-ix}$ and taking the inverse FT gives us our expression for the amplitude of the diffraction pattern.

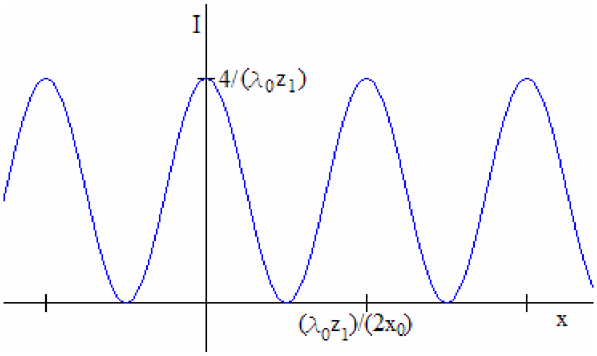
$$=\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+x_0{}^2\right)}\left(2\cos\left(\frac{\pi}{\lambda_0 z_1}2xx_0\right)\right)\left(\sqrt{\lambda_0 z_1}e^{\frac{i\pi}{4}}\right)$$

$$=\frac{2}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+x_0{}^2\right)}\cos\left(\frac{\pi}{\lambda_0 z_1}2xx_0\right)$$

We can now solve for the irradiance of the pattern.

$$I=|A|=\left|\frac{2}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+x_0{}^2\right)}\cos\left(\frac{\pi}{\lambda_0 z_1}2xx_0\right)\right|$$

$$=\frac{4}{\lambda_0 z_1}\cos^2\left(\frac{\pi}{\lambda_0 z_1}2xx_0\right)$$



(b)

$$f[x,y;0]*h[x,y;z_1]$$

$$=\left(RECT\left[\frac{x+x_0}{b}\right]+RECT\left[\frac{x-x_0}{b}\right]\right)\cdot 1[y]*\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)}$$

The problem can be simplified by rewriting the two RECT functions as:

$$=\left(RECT\left[\frac{x}{2(x_0+b)}\right]-RECT\left[\frac{x}{2(x_0-b)}\right]\right)\cdot 1[y]*\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)}$$

As discussed in the notes the rectangular aperture can be defined using two step functions.

$$=\left(\left(STEP[x+(x_0+b)]-STEP[x-(x_0+b)]\right)\cdot 1[y]*\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)}\right)$$
$$-\left(\left(STEP[x+(x_0-b)]-STEP[x-(x_0-b)]\right)\cdot 1[y]*\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)}\right)$$

Using results shown in the section of the notes title “Fresnel Diffraction from a Knife Edge” we have the following:

$$=\frac{1}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(\left(\int_{-\infty}^{x+(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x-(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha\right)-\left(\int_{-\infty}^{x+(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x-(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha\right)\right)$$

$$=\frac{1}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(\int_{-\infty}^{x+(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x-(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x+(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha+\int_{-\infty}^{x-(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha\right)$$

The associated irradiance is the time average of the squared magnitude:

$$I[x,y;z_1]=\left\langle\left|f[x,y;0]*h[x,y;z_1]\right|^2\right\rangle$$

$$=\left\langle\left|\frac{1}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(\int_{-\infty}^{x+(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x-(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x+(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha+\int_{-\infty}^{x-(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha\right)\right|^2\right\rangle$$

$$=\frac{1}{\lambda_0 z_1}\left|\left(\int_{-\infty}^{x+(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x-(x_0+b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x+(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha+\int_{-\infty}^{x-(x_0-b)}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha\right)\right|^2$$

6.

As in problem 5b, the rectangular aperture can be defined using two step functions. We once again define the diffraction pattern by convolving with the impulse response.

$$f[x,y;0]*h[x,y;z_1]$$
$$=\left(STEP\left[x+\frac{b}{2}\right]-STEP\left[x-\frac{b}{2}\right]\right)1[y]*\frac{1}{i\lambda_0 z_1}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}e^{\frac{i\pi}{\lambda_0 z_1}\left(x^2+y^2\right)}$$

Using results shown in the section of the notes title "Fresnel Diffraction from a Knife Edge" we have the following:

$$=\frac{1}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(\int_{-\infty}^{x+b/2}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha-\int_{-\infty}^{x-b/2}e^{\frac{i\pi\alpha^2}{\lambda_0 z_1}}d\alpha\right)$$

We want a similar result at a distance of $2z_1$ and an aperture of width d.

$$\frac{1}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(\int_{-\infty}^{x+d/2}e^{\frac{i\pi\alpha^2}{\lambda_0 2z_1}}d\alpha-\int_{-\infty}^{x-d/2}e^{\frac{i\pi\alpha^2}{\lambda_0 2z_1}}d\alpha\right)$$

We can match the integrals with those from our aperture of width b by making the substitutions: $\alpha=\sqrt{2}\beta$, $d\alpha=\sqrt{2}d\beta$. This will help us insure a matching diffraction pattern.

$$\frac{1}{\sqrt{i\lambda_0 z_1}}e^{2\pi i\left(\frac{z_1}{\lambda_0}-\nu t\right)}\left(\int_{-\infty}^{\left(1/\sqrt{2}\right)\left(x+d/2\right)}e^{\frac{i\pi\beta^2}{\lambda_0 z_1}}\sqrt{2}d\beta-\int_{-\infty}^{\left(1/\sqrt{2}\right)\left(x-d/2\right)}e^{\frac{i\pi\beta^2}{\lambda_0 z_1}}\sqrt{2}d\beta\right)$$

In order for the diffraction patterns to be scaled replics the integral in the two expressions need to have the same bounds. Each pattern has two integrals giving us two equation with two unknowns x, and d. We can then solve for d.

$$x+\frac{b}{2}=\frac{1}{\sqrt{2}}\left(x+\frac{d}{2}\right)\qquad x-\frac{b}{2}=\frac{1}{\sqrt{2}}\left(x-\frac{d}{2}\right)$$
$$x-\frac{x}{\sqrt{2}}=\frac{d}{2\sqrt{2}}-\frac{b}{2}\qquad x-\frac{x}{\sqrt{2}}=\frac{b}{2}-\frac{d}{2\sqrt{2}}$$
$$\frac{d}{2\sqrt{2}}-\frac{b}{2}=\frac{b}{2}-\frac{d}{2\sqrt{2}}$$
$$d=\sqrt{2}b$$