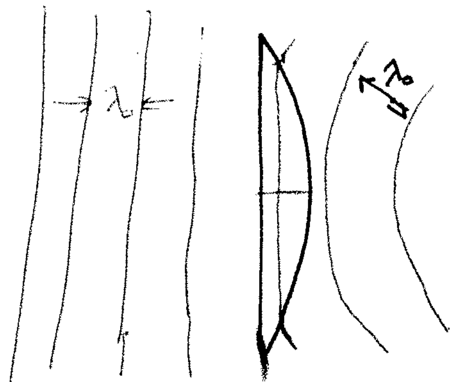
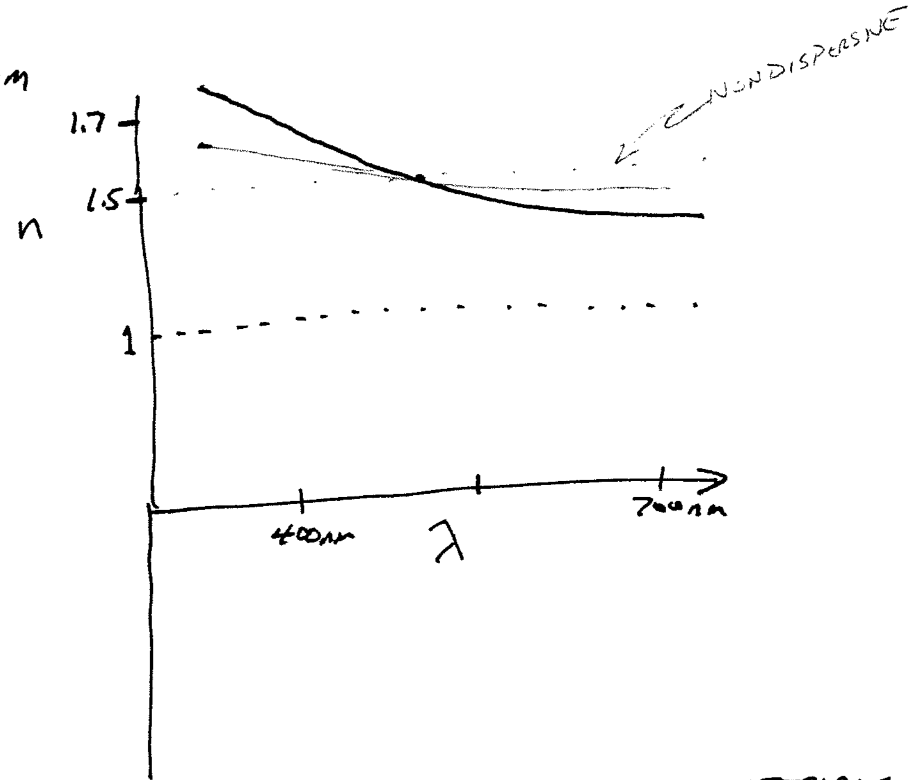
1/23/08

REFRACTIVE INDEX → DISPERSION

$n \equiv \frac{c}{v}$ v = VELOCITY IN MEDIUM





$n(\lambda) \rightarrow n(\nu)$

~~How~~?

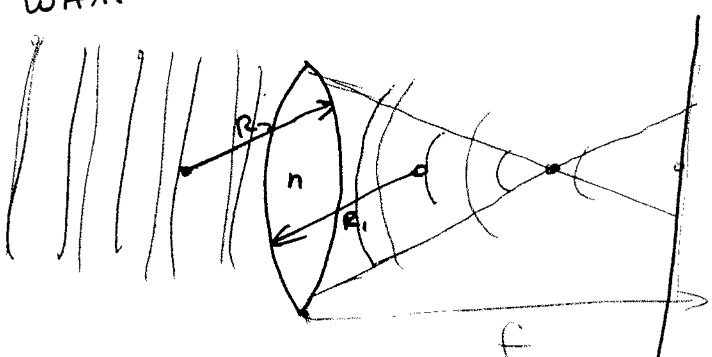
WHY?

WHAT IS VARIATION?

WHAT IS EFFECT ON IMAGE?

METAMATERIALS

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

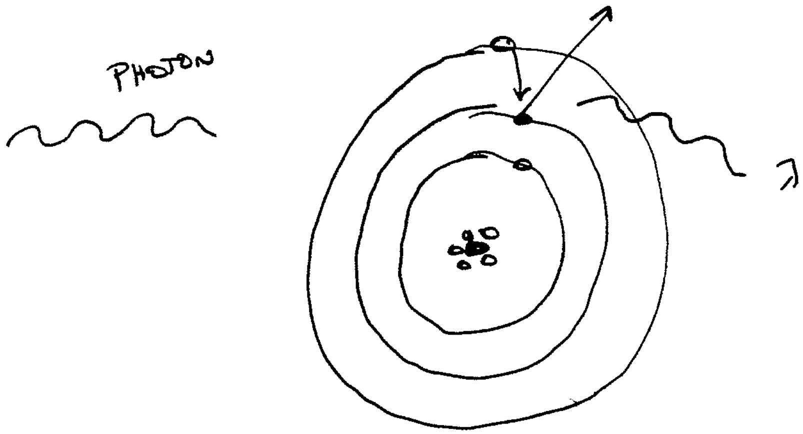


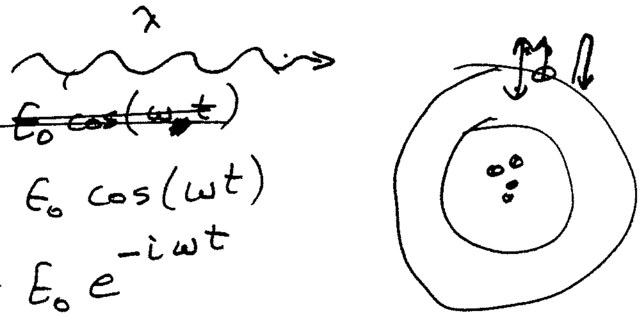
CHROMATIC ABERRATION

VARIATION IN IMAGE POSITION w/ WAVELENGTH

NORMAL DISPERSION As $\lambda \uparrow$ $n \downarrow$ $\frac{dn}{d\lambda} < 0$ NORMAL

ANOMALOUS DISPERSION As $\lambda \uparrow$ $n \uparrow$ $\frac{dn}{d\lambda} > 0$ ANOMALOUS





$m_p \simeq 1836\, m_e$

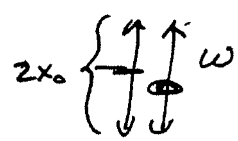
$\cancel{E = E_0 \cos(\omega_0 t)}$

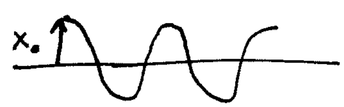
$E = E_0 \cos(\omega t)$

$= E_0 e^{-i\omega t}$

$F \propto e E[t] = e E_0 e^{-i\omega t}$

FEYNMAN LECTURES ON PHYSICS





FORCE OF MOTION eE

RESTORING FORCE – NATURAL OSCILLATOR

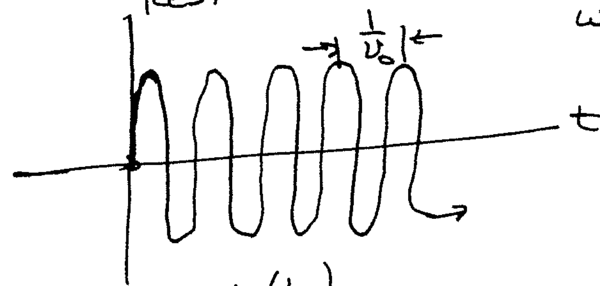
$h(t)$

RESPONSE OF ELECTRON TO DELTA FN. $g(t) = f(t) * h(t)$

$f(t)$

$*$

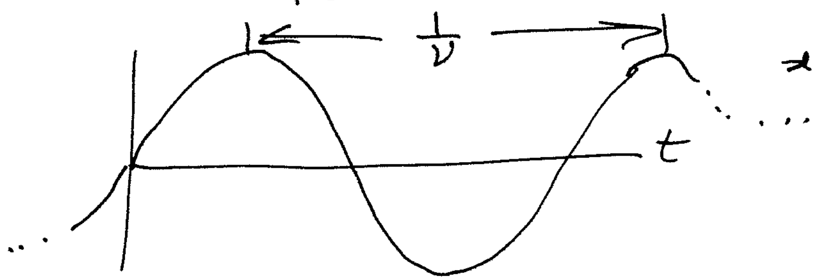
$\omega_0 = 2\pi\nu_0$



$h(t)$

$g(t)$

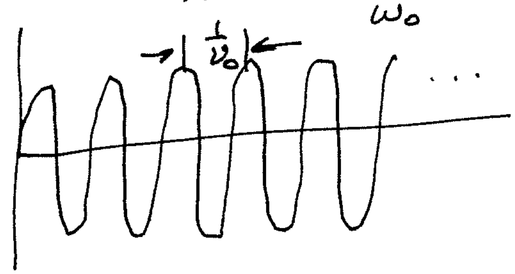
$f(t)$

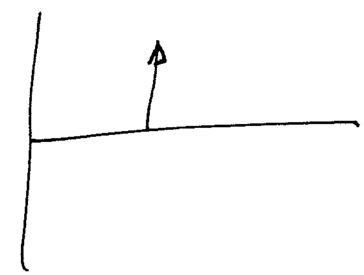


$*$

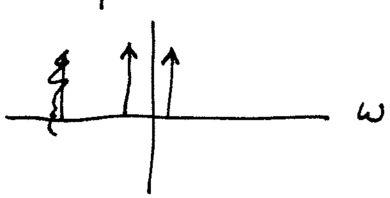
$h(t)$

ω_0

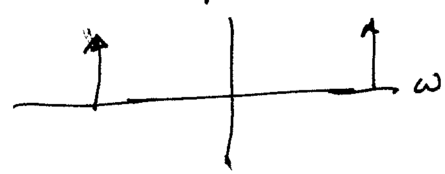




$|F(\omega)|\; e^{+i\pi x^2} * e^{-i\pi x^2} = \delta(x)$

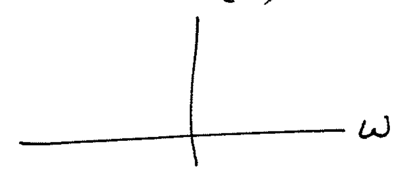


$|H(\omega)|$



$=$

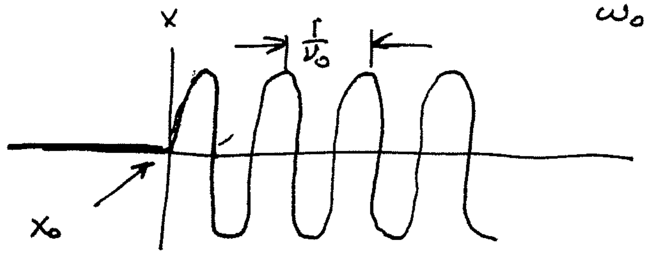
$O(\omega)$



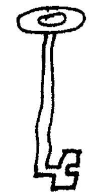
CAUSAL SYSTEM

$h(t) = 0$ FOR $t < 0$

$\omega_0 = 2\pi\nu_0$



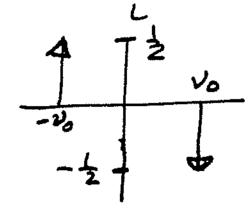
$$h(t) = \text{STEP}(t)\,\sin(\omega_0 t)$$



$$f(t) = E_0 \cos(\omega t) \quad \left(\cdot\ \text{STEP}(t)\right)$$

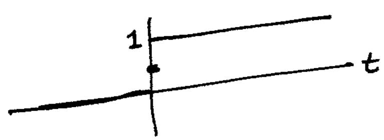
$$h(t) = \text{STEP}[t]\ \sin(2\pi\nu_0 t) \implies H(\nu) = \mathcal{F}\{\text{STEP}[t]\} * \mathcal{F}\{\sin(2\pi\nu_0 t)\}$$

$*$

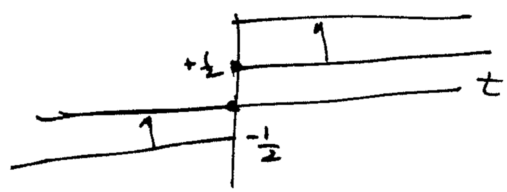


$\delta\delta\left(\frac{\nu}{\nu_0}\right)$

STEP(t)



$$\text{STEP}(t) = \underset{\uparrow}{\frac{1}{2}} + \frac{1}{2}\underset{\uparrow}{\underbrace{\text{SGN}(t)}}$$

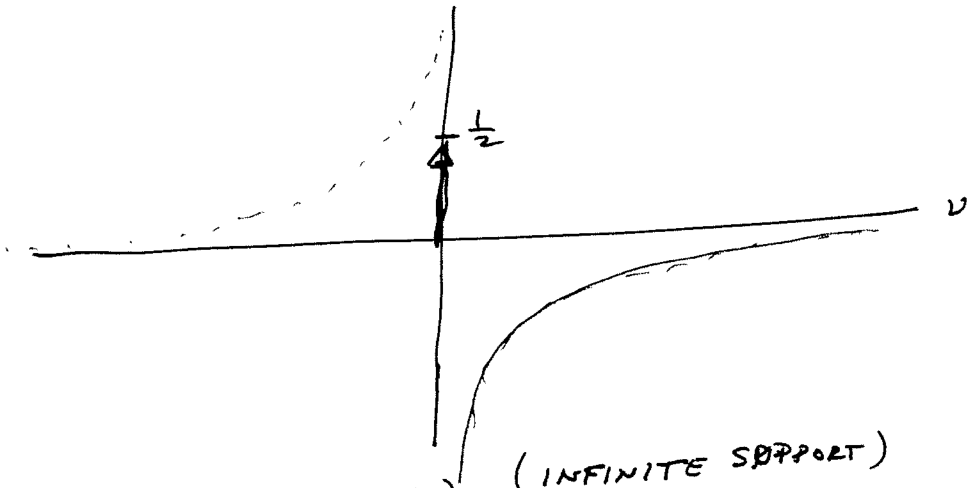


$$\text{STEP}[t] = \frac{1}{2}\cdot 1(t) + \frac{1}{2}\cdot \text{SGN}(t) = \frac{1}{2}\delta(\nu) + \frac{1}{2\pi i\nu} = \frac{1}{2}\delta(\nu) - i\left[\frac{1}{2\pi\nu}\right]$$

$$\longrightarrow \frac{1}{2}\delta(\nu) + \frac{1}{2}\cdot\frac{1}{i\pi\nu}$$

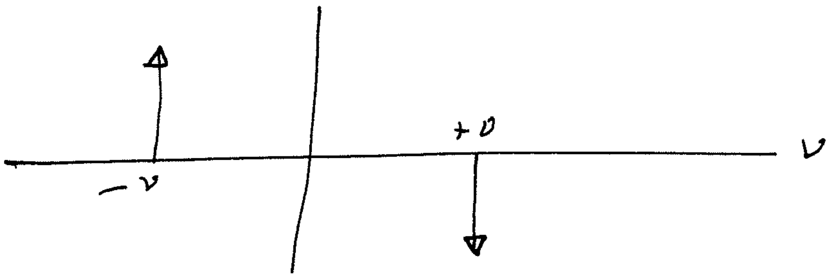
$\mathcal{F}\{\text{STEP}(t)\}$

——— REAL PART
- - - - IMAGINARY PART



(INFINITE SUPPORT)

F(ν)



5A

$$H(\nu) = \underbrace{\left[\frac{1}{2}\delta(\nu) + i\left(\frac{1}{2\pi\nu}\right)\right]}_{\text{STEP}} * \underbrace{i\left[\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right]}_{\text{SIN}}$$

Real Part

$$H[\nu] = \left[+1 \cdot \left(\frac{1}{2\pi\nu} * \delta(\nu+\nu_0)\right) - \left(\frac{1}{2\pi\nu} * \delta(\nu-\nu_0)\right)\right]$$

Imaginary Part

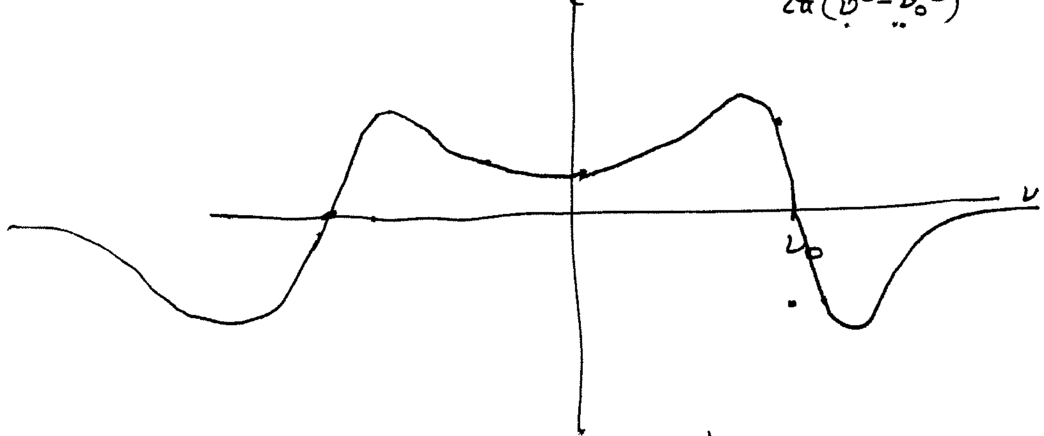
$$+ i \cdot \left[\frac{1}{2}\delta(\nu) * \left(\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right)\right]$$

$$= \left(\frac{1}{2\pi(\nu+\nu_0)} - \frac{1}{2\pi(\nu-\nu_0)}\right) + i\,\frac{1}{2}\left(\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right)$$

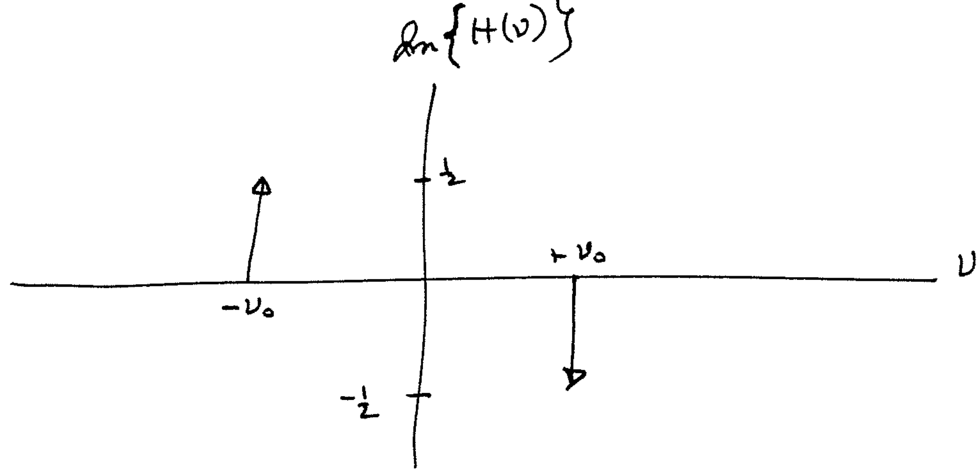
$$= \left(\frac{\nu-\nu_0}{2\pi(\nu+\nu_0)(\nu-\nu_0)} - \frac{(\nu+\nu_0)}{2\pi(\nu-\nu_0)(\nu+\nu_0)}\right) + i\,\frac{1}{2}\left[\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right]$$

$$= \frac{-2\nu_0}{2\pi(\nu^2-\nu_0^2)} + i \cdot \frac{1}{2}\left[\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right]$$

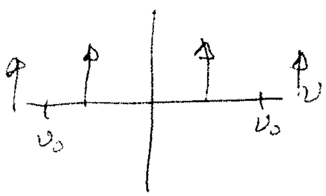
$$\mathrm{Re}\{H(\nu)\} = -\frac{\nu_0}{2\pi(\nu^2-\nu_0^2)}$$



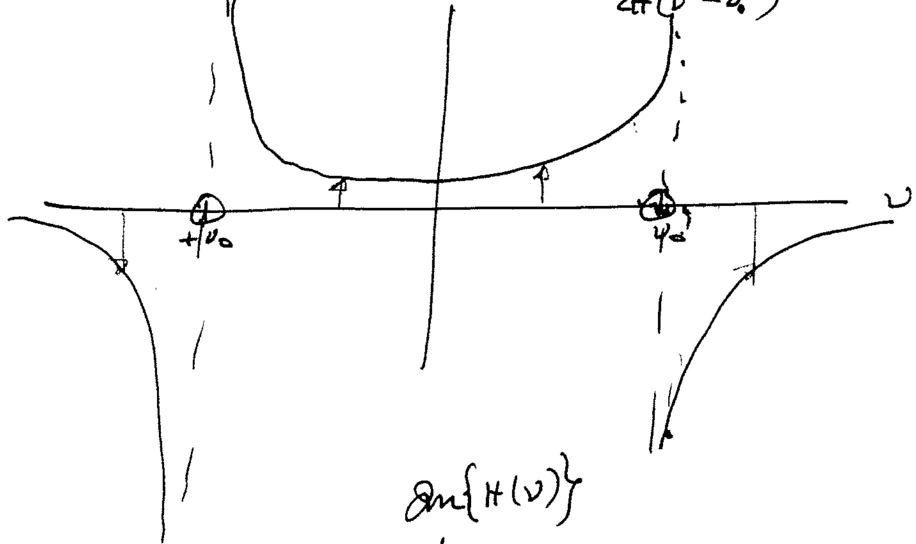
IF $\nu \gtrsim \nu_0 \Rightarrow \mathrm{Re}\{H(\nu)\} < 0$



$F(\nu)$



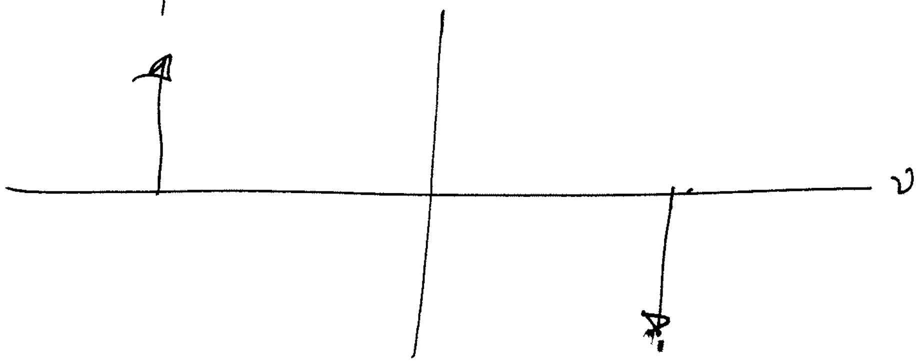
$$\mathrm{Re}(H(\nu)) = \frac{-c\nu_0}{2\pi(\nu^2-\nu_0^2)}$$



OSCILLATION FREQ OF ELECTRON

= OSC. FREQ. OF FIELD

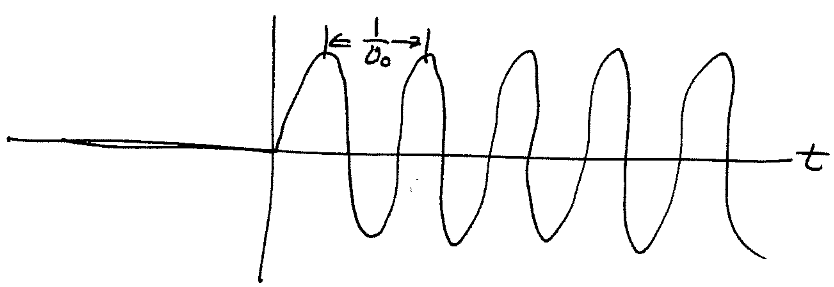
$\mathrm{Im}\{H(\nu)\}$



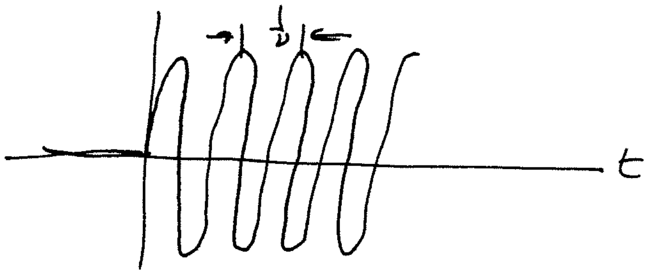
$$\alpha\,\delta(\nu-\nu_0)\cdot\beta\,\delta(\nu-\nu_0)$$

$$\alpha\beta\,\delta(\nu-\nu_0)$$

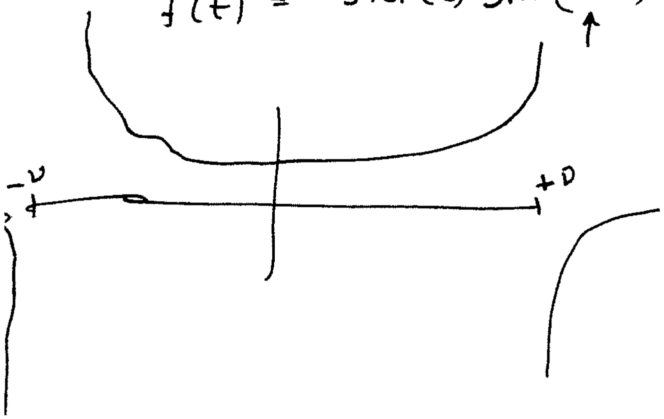
$h(t)$

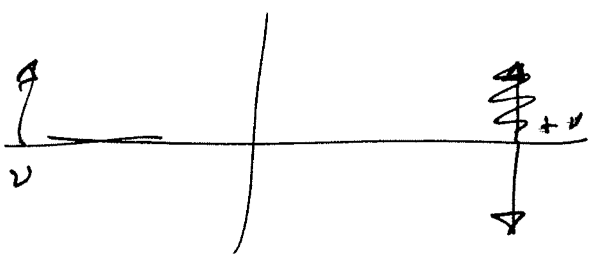


$f(t)$

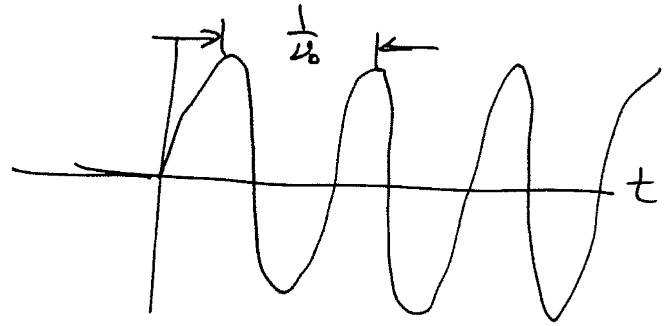


$f(t) = \mathrm{STEP}(t)\,\mathrm{SIN}(\omega t)$

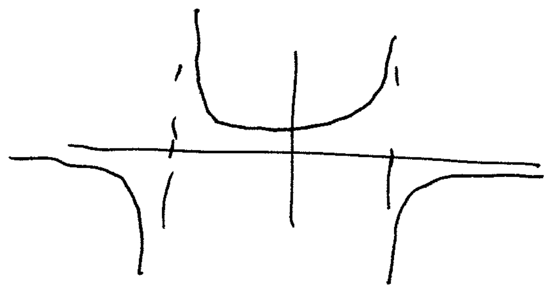


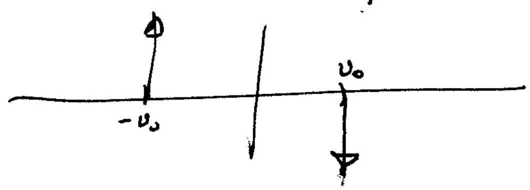


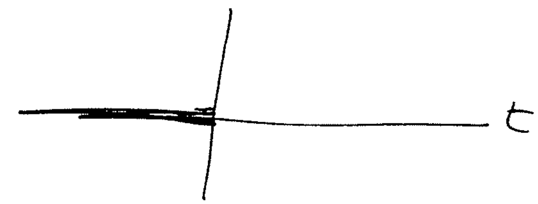
$h(t)$



$h(t) = \mathrm{STEP}(t)\,\mathrm{SIN}(\omega_0 t)$







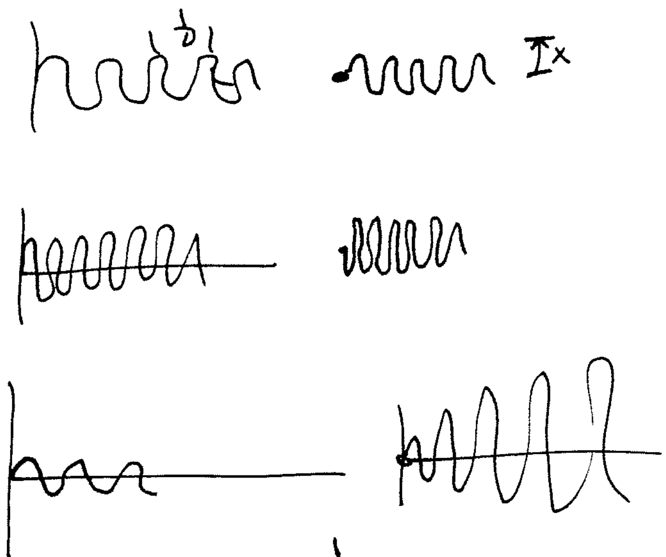
$g(t) = ?$

$= \mathrm{STEP}(t) \cdot ?$

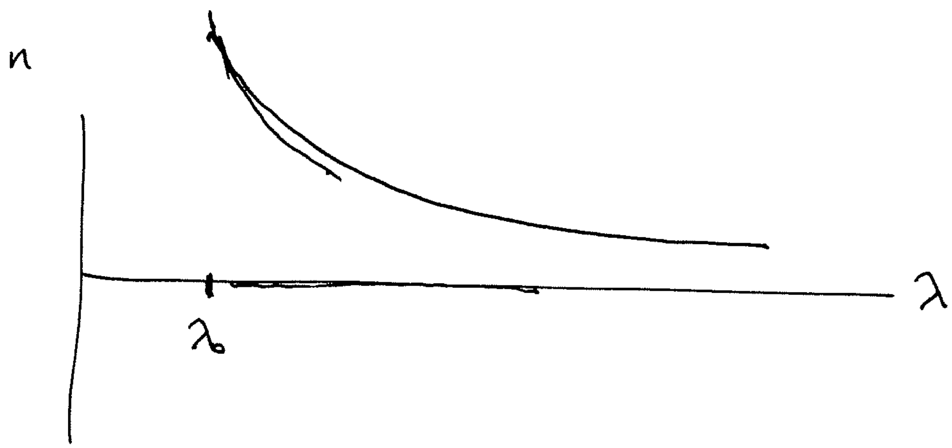
OUTPUT AMPLITUDE IS FUNCTION OF ν & ν_0

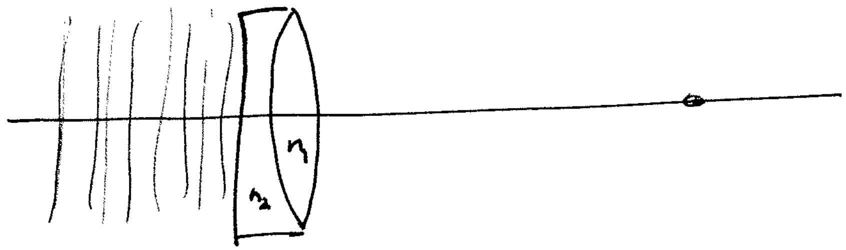
$\uparrow$ DRIVING FREQ

$\uparrow$ NATURAL FREQ (RESONANT FREQ)



$$x = f(\nu, \nu_0)$$





COLOR CORRECTED LENS

ACHROMAT

APOCHROMAT

MATERIAL PARAMETERS

PERMITTIVITY ϵ , ϵ_0 (PENETRATION INTO MATERIAL OF E)

PERMEABILITY μ μ_0 (" " " " B)

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$$

$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

$$\frac{c}{v} \equiv n = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}} \Rightarrow n = \sqrt{\frac{\epsilon}{\epsilon_0}} \Rightarrow n^2 = \frac{\epsilon}{\epsilon_0}$$

FOR US, $\mu_0 = \mu$

$$n^2 = \frac{\epsilon}{\epsilon_0}$$

IF ABSORPTION IN GLASS $\Rightarrow$ n IS COMPLEX VALUED

$$\tilde{n} = n + iK \qquad n(1 + i\chi)$$

$$V = \frac{\omega}{k} \qquad \left.\begin{aligned} \omega &= 2\pi\nu \\ k &= \frac{2\pi}{\lambda} \end{aligned}\right\} \quad v = \lambda \cdot \nu$$

IN VACUUM $\quad \lambda_0 , \nu_0 \; ; \quad \lambda_0 \cdot \nu_0 = c = \dfrac{\omega_0}{k_0}$

WAVE $\quad f(z,t) = e^{i(kz - \omega t)} \qquad V = \dfrac{\omega}{k} = \lambda \cdot \nu$

$$= e^{i\left(2\pi \frac{z}{\lambda} - 2\pi \nu t\right)}$$

IN VACUUM $\quad \lambda_0 , \nu_0 , \omega_0 , k_0 \qquad e^{i(k_0 z - \omega_0 t)} = e^{i\left(2\pi \frac{z}{\lambda_0} - 2\pi \nu_0 t\right)}$

IN MATERIAL $\quad V = \dfrac{c}{n}$

RECALL: $\quad E = h\nu \quad$ FOR LIGHT

IF ENERGY IS CONSERVED $\quad \nu_0 = \nu$

$$\left.\begin{aligned} V &= \lambda \cdot \nu \\ c &= \lambda_0 \cdot \nu_0 \end{aligned}\right\} \quad \nu = \nu_0 \qquad \begin{aligned} V &= \lambda \nu_0 \\ c &= \lambda_0 \nu_0 \end{aligned} \qquad \lambda \neq \lambda_0$$

$$\frac{c}{V} = n = \frac{\lambda_0}{\lambda} \Rightarrow \lambda \cdot n = \lambda_0 \quad ; \quad n \geq 1$$

$$\lambda = \frac{\lambda_0}{n} < \lambda_0$$