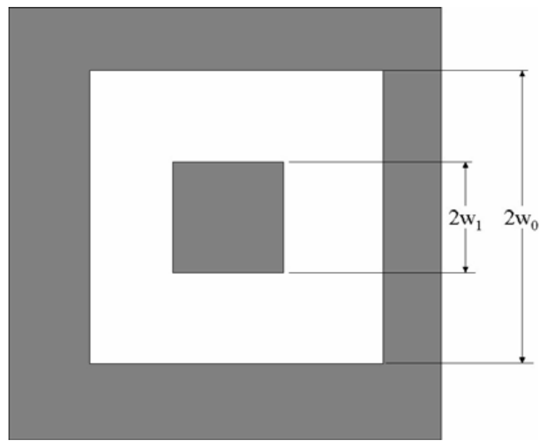
1051-733-20072 Homework #3 Due 1/9/2008 (W)

1. For the aperture distribution shown, where the aperture is square and has a square central obscuration, and "white" represents transparent regions and "gray" represents opaque, and assuming normally incident monochromatic plane-wave illumination with λ_0:

   (a) Find an expression for the "intensity" (i.e., irradiance) distribution for light that has propagated a distance z_1 into the Fraunhofer diffraction region

   (b) Sketch the x-profile of this distribution for the propagation distance.



2. Consider a diffraction grating whose transmittance is 0 or 1 over regions with identical widths d_0 (as we did in class). The grating is then "windowed" (or "apodized") by a rectangle function $RECT\left[\frac{x}{b_0}, \frac{y}{b_0}\right]$.

   (a) Find the expression for the light diffracted by this windowed grating in the Fraunhofer diffraction region as a function of λ_0, z_1, b_0, and d_0.

   (b) Describe what happens using equations and sketches if the grating is illuminated by a source with two equal-strength wavelengths λ_1 and $\lambda_2 > \lambda_1$.

3. We said that a spherical lens may be approximated as a multiplicative quadratic phase:

$$t_{\text{spherical}}[x, y] = p[x, y] \cdot \exp\left[-i\pi\frac{x^2 + y^2}{\lambda_0 \mathbf{f}}\right]$$

   (a) What is the action of a multiplicative phase function with the form:

$$t_1[x, y] = p[x, y] \cdot \exp\left[-i\pi\frac{x^2}{\lambda_0 \mathbf{f}}\right]$$

(b) Find the distribution of light from an object $f[x, y]$ generated by this lens if placed in an optical imaging system at a large distance z_1 from the object so that the image is formed at the approximate distance $z_2 \gtrapprox \mathbf{f}$. In other words, how does the distribution of light at the image relate to the object function $f[x, y]$?

(c) Describe the image distribution that is generated if an identical lens is added to the system at right angles to the first and touching it. In other words, the transmittance of the second lens is:

$$t_2[x, y] = p[x, y] \cdot \exp\left[-i\pi \frac{y^2}{\lambda_0 \mathbf{f}}\right]$$

so that

$$t[x, y] = t_1[x, y] \cdot t_2[x, y]$$

4. Consider a grating whose transmittance is a sinusoidal function of x:

$$f[x, y] = \frac{1}{2} + \frac{\alpha}{2}\cos\left[2\pi \frac{x}{L}\right] \cdot 1[y]$$

(a) Use the transfer function of light propagation in the Fresnel diffraction region to find the spectrum of the Fresnel diffraction pattern at some propagation distance z_1.

(b) Find a relationship for the propagation distance z_1 that produces replicas of the amplitude of the original grating function, even though the system includes no lenses. Your relationship should indicate that the same result is obtained for many propagation distances z_1, which means that the periodic grating forms images of itself at various distances in the Fresnel diffraction region. These are called *Talbot images*, after William Henry Fox Talbot, who first explained them. Note that Talbot images are produced for any periodic object, not just for gratings.

(c) Find the formula for the distances where the images of the grating exhibit reversed contrast, i.e., what was a transmissive region appears as an opaque region, and vice versa.

5. Consider the two-dimensional chirp-function transmittance:

$$f[x] = \frac{1}{2} + \frac{1}{2}\cos\left[\pi \frac{x^2 + y^2}{\alpha^2}\right]$$

where α has units of length. (this is called a "Fresnel zone plate" if the amplitude is thesholded at $f = \frac{1}{2}$ so that larger transmittances are mapped to "1" and smaller transmittances to "0"). If $\alpha = 1\,\text{mm}$ and the object is illuminated with plane waves with wavelength $\lambda_0 = 632.8\,\text{nm}$ (He:Ne laser), evaluate the Fresnel diffraction pattern of this transmittance. Show that transmittance pattern acts as a lens with three different focal lengths and determine them.

$$\text{(unnecessary hint): } \cos[\theta] = \frac{\exp[+i\theta] + \exp[-i\theta]}{2}$$