Finish reading §3 *Optical Diffraction and Imaging*

1. Find AND GRAPH the irradiance distribution (squared magnitude of the amplitude) of the Fresnel diffraction pattern on the optical axis (i.e., a function of \( z \) instead of a function of \([x, y]\)) if \( f[x, y] = CYL(r) \) (HINT: write down the Fresnel diffraction formula as integral and change variable to more convenient form).

2. The Fresnel diffraction pattern from a rectangular aperture of width \( b_0 \) propagated over the distance \( z_1 \) is \( g[x, y] \). Find the width \( d_0 \) of the aperture in the same plane as the original aperture that produces a scaled replica of \( g[x, y] \) if the propagation distance is \( 2 \cdot z_1 \).

3. For the aperture distribution shown, where the aperture is square and has a square central obscuration, and “white” represents transparent regions and “gray” represents opaque, and assuming normally incident monochromatic plane-wave illumination with \( \lambda_0 \):
   
   (a) Find an expression for the “intensity” (i.e., irradiance) distribution for light that has propagated a distance \( z_1 \) into the Fraunhofer diffraction region.
   
   (b) Sketch the \( x \)-profile of this distribution for the propagation distance.
4. A diffracting screen has a circularly symmetric amplitude transmittance function given by:

\[ t_1 (r) = \frac{1}{2} \left( 1 + \cos \left[ \beta r^2 \right] \right) \cdot CYL \left( \frac{r}{d_0} \right) \]

where \( \beta \) is a constant and \( CYL (r) \) is defined as before:

\[
CYL (r) \equiv \begin{cases} 
1 & \text{for } r < \frac{1}{2} \\
\frac{1}{2} & \text{for } r = \frac{1}{2} \\
0 & \text{for } r > \frac{1}{2} 
\end{cases}
\]

(a) (zzzz) What are the dimensions of \( \beta \)?
(b) Sketch \( t_1 (r) \) for the case where \( d_0 \) is chosen so that the largest phase angle in the cosine is \( 16\pi \) radians
(c) In what ways does this screen act like a lens? (HINT: Euler relation).
(d) Sketch the transmittance after thresholding at \( t_0 = \frac{1}{2} \):

\[ t_2 (r) = \frac{1}{2} \left( 1 + \text{SGN} \left[ \cos \left[ \beta r^2 \right] \right] \right) \cdot CYL \left( \frac{r}{d_0} \right) \]

(e) Describe the qualitative difference in performance in the systems using \( t_1 (r) \) and \( t_2 (r) \).

5. Consider a grating whose transmittance is a sinusoidal function of \( x \):

\[ f [x, y] = \left( \frac{1}{2} + \frac{\alpha_0}{2} \cos \left[ \frac{2\pi}{L_0} \frac{x}{x} \right] \right) \cdot 1 [y] \]

where \( \alpha_0 \leq 1 \).

(a) Use the transfer function of light propagation in the Fresnel diffraction region to find the spectrum of the Fresnel diffraction pattern at some propagation distance \( z_1 \).
(b) Find a relationship for the propagation distances \( z_1 \) that produce replicas of the amplitude of the original grating function, even though the system includes no lenses. Your relationship should demonstrate that the same result is obtained for many propagation distances \( z_1 \), which means that the periodic grating forms images of itself at various distances in the Fresnel diffraction region. These are called Talbot images, after William Henry Fox Talbot, who first explained them. Note that Talbot images are produced for any periodic object, not just for gratings.