1051-733-20072 Homework #2 Due 12/19/2007 (W)

1. Show that the volume of the quadratic-phase impulse response for Fresnel diffraction without the constant phase is unity

$$h[x,y;z_1,\lambda_0] = \iint_{-\infty}^{+\infty} \left(\frac{1}{i\lambda_0 z_1} \exp\left[+i\frac{\pi}{\lambda_0 z_1}\left(x^2+y^2\right)\right]\right) dx\ dy = 1$$

2. Consider propagation over the distance z_1 and then over the distance z_2, where both distances satisfy the conditions for Fresnel diffraction. Show that a single propagation over the distance $z_1 + z_2$ gives the same result as the propagations over z_1 and then over z_2.

3. Consider a spherical wave expanding about the point $[0, 0, -z_1]$ in a Cartesian coordinate system. The wavelength of the light is λ_0 and the position $z_1 > 0$.

   (a) Express the phase distribution of the spherical wave across the $[x, y]$ plane located normal to the z-axis at coordinate $z = 0$.

   (b) Use the paraxial approximation to find an expression for the phase distribution of the parabolic wavefront (quadratic-phase factor) that approximates this spherical wavefront.

   (c) Find an exact expression for the phase by which the spherical wavefront *lags* or *leads* the phase of the parabolic waefront. Does it lag or lead? A (relatively accurate) sketch will definitely help.

4. A spherical wave converges towards the point $[0, 0, z_0]$ to the right of a circular aperture of radius R centered at $[0, 0, 0]$ (so that $z_0 > 0$). The wavelength of the light is λ_0. The light is observed at an arbitrary location $[x, y]$ at axial location specified by z to the right of the aperture. Show that the wavefront error made in a paraxial approximation of the illuminating converging spherical wave and the error incurred using the quadratic-phase approximation in the Fresnel diffraction equation (that represents a diverging parabolic wave) partially cancel one another. Under what condition does complete cancellation occur?

5. Derive expressions for the amplitude and the irradiance of the Fresnel diffraction patterns observed in light with wavelength λ_0 at the plane $z = z_1$ for the following source distributions:

   (a) $f[x,y;0] = (\delta[x+x_0] + \delta[x-x_0]) \cdot 1[y]$ (SKETCH the diffraction pattern)

   (b) $f[x,y;0] = \left(RECT\left[\frac{x+x_0}{b}\right] + RECT\left[\frac{x-x_0}{b}\right]\right) \cdot 1[y]$ where $x_0 > b > 0$

6. For Fresnel diffraction from a rectangular aperture of width b propagated over the distance z_1, find the width d of the aperture in the same plane that produces a scaled replica of the same pattern if the propagation distance is $2z_1$.