1051-733-20092 Homework #1 Due 12/09/2000 (W)

Read Notes §1 Introduction, Skim §2 Wave Optics and Imaging, start to read §3 Optical Diffraction and Imaging

Warmup Problems

1. The speed of light in vacuum is approximately $3 \times 10^8 \text{ m/s}$. Find the wavelength of light having a frequency of 1 PHz. Compare this to the wavelength of a 60 Hz electromagnetic wave.

2. (HW1-20072-2) The figure shows the profile of a transverse wave on a string traveling in the positive $z$-direction at a speed of $1 \text{ m/s}$.

   (a) Determine its spatial period.
   (b) Notice that as the wave passes any fixed point on the $z$-axis, the string at that location oscillates in time. Draw a graph of $\psi [t]$ showing how a point on the rope at $z = 0$ oscillates.
   (c) Determine the temporal frequency of the wave.

3. A harmonic traveling wave is moving in the negative $z$-direction with an amplitude of 2 arbitrary units, a wavelength of 5 m, and a period of 3 s. Its displacement at $z = 0$ and $t = 0$ is zero. Write a wave equation for this wave under the following conditions:

   (a) explicitly in terms of both wavelength and period
   (b) exhibits both propagation constant and velocity
   (c) in complex form
Diffraction problems

4. A hole with diameter $d_0 = 1$ mm is illuminated by light with $\lambda_0 = 546$ nm. Determine which approximation (Fresnel or Fraunhofer) is valid for observation planes located at the distances: $z_1 = 500$ mm, $z_2 = 1$ m, and $z_3 = 5$ m.

5. For the quadratic-phase impulse response for Fresnel diffraction without the constant phase:

$$ h [x, y; z_1, \lambda_0] = \frac{1}{i \lambda_0 z_1} \exp \left[ +i \frac{\pi}{\lambda_0 z_1} \left( x^2 + y^2 \right) \right] $$

(a) Show that the volume is unity.

(b) Show that the two-dimensional sinusoidal part of this function contributions ALL of the volume and that the cosine part contributes none.

6. Consider propagation over the distance $z_1$ and then over the distance $z_2$, where both distances satisfy the conditions for Fresnel diffraction. Show that a single propagation over the distance $z_1 + z_2$ gives the same result as the propagations over $z_1$ and then over $z_2$.

7. Consider a spherical wave expanding about the point $[0, 0, -z_1]$ in a Cartesian coordinate system. The wavelength of the light is $\lambda_0$ and $z_1 > 0$.

(a) Express the phase distribution of the spherical wave across the $[x, y]$ plane located normal to the $z$-axis at coordinate $z = 0$.

(b) Use the paraxial approximation to find an expression for the phase distribution of the parabolic wavefront (quadratic-phase factor) that approximates this spherical wavefront.