1051-733-20072 Homework #1 Due 12/10/2007 (M)

Read Notes §1 *Wave Motion* and §7 *Superposition of waves.* Read §3.1-3.3 *Electromagnetic Theory* and start on §4 *Propagation of Light*

Warmup Problems

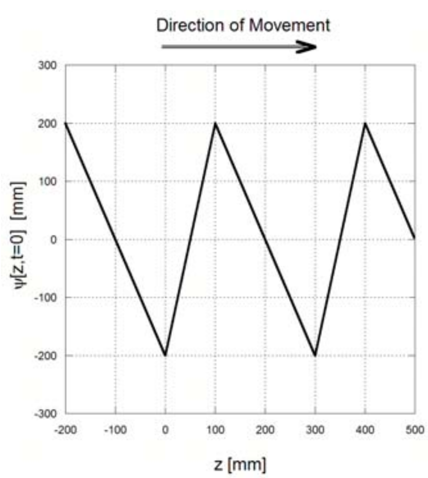
1. The speed of light in vacuum is approximately $3 \times 10^8 \frac{\text{m}}{\text{sec}}$. Find the wavelength of red light having a frequency of 5×10^{14} Hz. Compare this to the wavelength of a 60 Hz electromagnetic wave.

2. The figure shows the profile of a transverse wave on a string traveling in the positive z-direction at a speed of $1 \frac{\text{m}}{\text{s}}$.

   (a) Determine its spatial period.

   (b) Notice that as the wave passes any fixed point on the z-axis, the string at that location oscillates in time. Draw a graph of $\psi[t]$ showing how a point on the rope at $z = 0$ oscillates.

   (c) Determine the temporal frequency of the wave.



3. (P[3] 4-5) A harmonic traveling wave is moving in the negative z-direction with an amplitude of 2 arbitrary units, a wavelength of 5 m, and a period of 3 s. Its displacement at $z = 0$ and $t = 0$ is zero. Write a wave equation for this wave under the following conditions:

   (a) explicitly in terms of both wavelength and period

   (b) exhibits both propagation constant and velocity

   (c) in complex form

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4. (P^3 4-5)

   (a) Write the equation of a harmonic wave traveling along the x-direction at $t = 0$ if it has an amplitude of 5 m and a wavelength of 50 m.

   (b) Write an expression for the disturbance at $t = 4\,\mathrm{s}$ if it is moving in the negative x-direction at $2\,\frac{\mathrm{m}}{\mathrm{s}}$.

5. (variant of P^3 4-14) Two waves of the same amplitude, speed, and frequency travel together in the same region of space. The resultant wave may be written as a sum of the individual waves:

$$\psi\left[z,t\right] = A\sin\left[k_0 z + \omega_0 t\right] + A\sin\left[k_0 z - \omega_0 t + \phi_0\right]$$

   Use complex exponentials to find an expression as the product of two sinusoidal functions:

6. Determine which of the following describe traveling waves. Where appropriate, draw the profile and find the speed and direction of motion.

   (a) $\psi\left[y,t\right] = \exp[[-\left(a^2y^2 + b^2t^2 - 2abty\right)]$

   (b) $\psi\left[z,t\right] = A\sin\left[az^2 - bt^2\right]$

   (c) $\psi\left[x,t\right] = A\sin\left[2\pi\left(\frac{x}{a} + \frac{t}{b}\right)^2\right]$

   (d) $\psi\left[x,t\right] = A\cos^2\left[2\pi\left(t - x\right)\right]$

7. The time average of some function $f\left[t\right]$ over the time interval T may be defined:

$$\langle f\left[t\right]\rangle \equiv \frac{1}{T}\int_t^{t+T} f\left[t'\right]dt'$$

   where t' is the dummy variable of integration. If $\tau = \frac{2\pi}{\omega_0}$ is the period of a harmonic function and $T = \tau$ or $T >> \tau$, show that:

$$\begin{aligned}
\left\langle \sin^2\left[\underline{\mathbf{k}}_0 \bullet \underline{\mathbf{r}} - \omega_0 t + \phi_0\right]\right\rangle &= \frac{1}{2} \\
\left\langle \cos^2\left[\underline{\mathbf{k}}_0 \bullet \underline{\mathbf{r}} - \omega_0 t + \phi_0\right]\right\rangle &= \frac{1}{2} \\
\langle \sin\left[\underline{\mathbf{k}}_0 \bullet \underline{\mathbf{r}} - \omega t + \phi_0\right]\cos\left[\underline{\mathbf{k}}_0 \bullet \underline{\mathbf{r}} - \omega_0 t + \phi_0\right]\rangle &= 0
\end{aligned}$$