SIMG-733 Optics Final Exam Closed Book

THREE HOURS (though targeted for two hours)
NO CALCULATORS!!
DO #1 (40%) and select ANY THREE (3) PROBLEMS from #2 - #6 (weighted equally at 20% each even if not equally difficult)
Submit problems stapled together **IN NUMERICAL ORDER**
STANDARD HINT: MAKE SKETCHES BEFORE WRITING EQUATIONS

1. An optical system consists of two lenses with the same focal length $\mathbf{f} = +100$ mm and separated by $t = +400$ mm. The first lens has "semidiameter" $r = +20$ mm (i.e., the diameter of the lens is $d = +40$ mm). An object of height $h = +20$ mm is located +200 mm to the left of the first lens.

   (a) Sketch the system showing the lenses and the object.

   (b) Cast out a ray from the "bottom" of the object that just "grazes" the edge of the first lens (this is the *marginal ray* of the system). Trace that ray through the rest of the system to determine the location where that ray crosses the optical axis after the second lens. This is the location of the image.

   (c) Cast out a ray from the top of the object that passes through the center of the first lens (this is the provisional *chief ray* of the system). Find the "height" of this ray at the output image, which determines the transverse magnification of the image.

   (d) Use the result of (c) to determine the "semidiameter" of the second lens that would be necessary for the ray in (c) to reach the image.

   (e) Now place a lens identical to the first lens (i.e., focal length $\mathbf{f} = +100$ mm and diameter $d = +40$ mm) midway between the two lenses. Cast out the same two rays from the same object and trace them through the system. Determine the location and the transverse magnification of the resulting image.

   (f) Determine the semidiameter of the last lens (the second lens in the first system) that would be required to pass the second ("chief") ray.

   (g) (Optional, Extra Credit) Explain the role of the third lens (between the first two).

2. An object consisting of two monochromatic point sources at wavelength λ_0:

$$f[x,y;0] = \delta[x,y] + \frac{1}{10}\delta[x - 10\,\text{mm}, y]$$

This light propagates a distance z_1 into the Fraunhofer diffraction region, where the amplitude pattern is $g[x,y;z_1]$.

(a) Determine the form of and SKETCH the observed irradiance pattern at $z = z_1$; you may ignore any leading constants.

(b) A photographic plate is exposed to this irradiance pattern and developed into a transparency with transmittance $t[x,y]$ proportional to the *complement* of the normalized irradiance, i.e., the transmittance is:

$$t[x,y;z_1] = 1 - \frac{\left\langle |g[x,y;z_1]|^2 \right\rangle}{\left\langle |g[x,y;z_1]|^2 \right\rangle_{\max}}$$

Write down the form of $t[x,y;z_1]$.

(c) The developed photographic plate is replaced at the same location (at the distance z_1 from the origin) and illuminated by the point source at the origin $\delta[x,y]$. The light from this source is diffracted by the transparency and propagates a distance z_2 to an observation plane in the Fraunhofer diffraction region. Compute the irradiance at this location. observation plane.

3. A plane wave with wavelength λ_0 propagates down the z-axis and illuminates a 2-D "object" of the form:

$$f[x,y;z=0] = \left(RECT\left[\frac{x+2d_0}{d_0}\right] + RECT\left[\frac{x-2d_0}{d_0}\right]\right) \cdot 1[y]$$

The light through the aperture then propagates further down the z-axis to an observation plane at $z = z_1$ in the Fresnel diffraction region.

(a) Sketch a profile along the x-axis of the diffraction pattern at the plane z_1; I'm looking for **qualitative** features, not *quantitative* ones.

(b) Now assume that the light propagates farther down the z-axis to the plane at $z = z_2$ in the Fraunhofer diffraction region. Find expressions for the amplitude and irradiance distributions of light at this plane.

(c) Sketch the irradiance distribution that was derived in part (b).

4. A lens with focal length $\mathbf{f}$ and diameter $d = +100\,\text{mm}$ is used to image an object emitting quasimonochromatic light centered about wavelength $\lambda_0 = 500\,\text{nm}$ and located at the distance $z_1 = +1\,\text{km}$.

   (a) Manipulate the imaging equation $z_1^{-1} + z_2^{-1} = \mathbf{f}^{-1}$ to find an expression for the transverse magnification in terms of the focal length $\mathbf{f}$. Use this to determine the effect of increasing the focal length on the transverse magnification.

   (b) Compare the resulting images of this object at $z_1 = +1\,\text{km}$ that would be produced by two lenses with diameter $d = +100\,\text{mm}$. The first lens has focal length $\mathbf{f}_1 = +500\,\text{mm}$ and the second has $\mathbf{f}_2 = +1000\,\text{mm}$.

5. Describe an optical system that could be used to measure the refractive index of some molecular gas as a function of the number density of the molecules over the range from vacuum to some high pressure. Include a sketch of the system, an explanation of how it would work, and information about the result that would be expected from the measurement.

6. A monochromatic star with $\lambda_0 = 460\,\text{nm}$ is setting over a smooth ocean surface. Assume that we are located at the equator and the star is on the celestial equator, so that the star path is perpendicular to the ocean surface. Some of the starlight travels directly to the observer and some is reflected from the surface. Describe qualitatively and quantitatively what is observed at the observation plane as a function of both space and time; include a sketch of the pattern observed at one time and indicate what happens as the star approaches the horizon. Possibly useful information: the earth rotates 360° in 24 hours, so the star appears to move 15° in one hour.