

1 Convolution Concepts

1.1 First Week: 1-D convolution using areas and optics, filtering of time signals

Convolution is the mathematical description of the action of a particular type of linear system that may be described completely by a function with the same dimensionality as the input function.

For a 1-D system described by the operator \mathcal{O} acting on the input function $f[x]$, the mathematical description is:

$$\mathcal{O}\{f[x]\} = \int_{-\infty}^{+\infty} f[\alpha] h[x - \alpha] d\alpha = g[x]$$

where $h[x, y]$ is the so-called “impulse response” or “point spread function of the system

Implementation of this process requires four mathematical operations:

1. reversal of $h[\alpha]$ (“flipping”)
2. translation of $h[-\alpha]$ to “center” it at coordinate x measured relative to $f[\alpha]$
3. multiplication of the two functions
4. integration (summation) to evaluate the area of the product

“Reversal” is accomplished by a simple “flipping” of the h transparency, translation by manually shifting the h transparency relative to f , multiplication by overlaying the two bitonal transparencies, and integration by summing the transmitted light. You will be making “relative” measurements only; further calibration would be necessary to convert the measurements to absolute numbers.

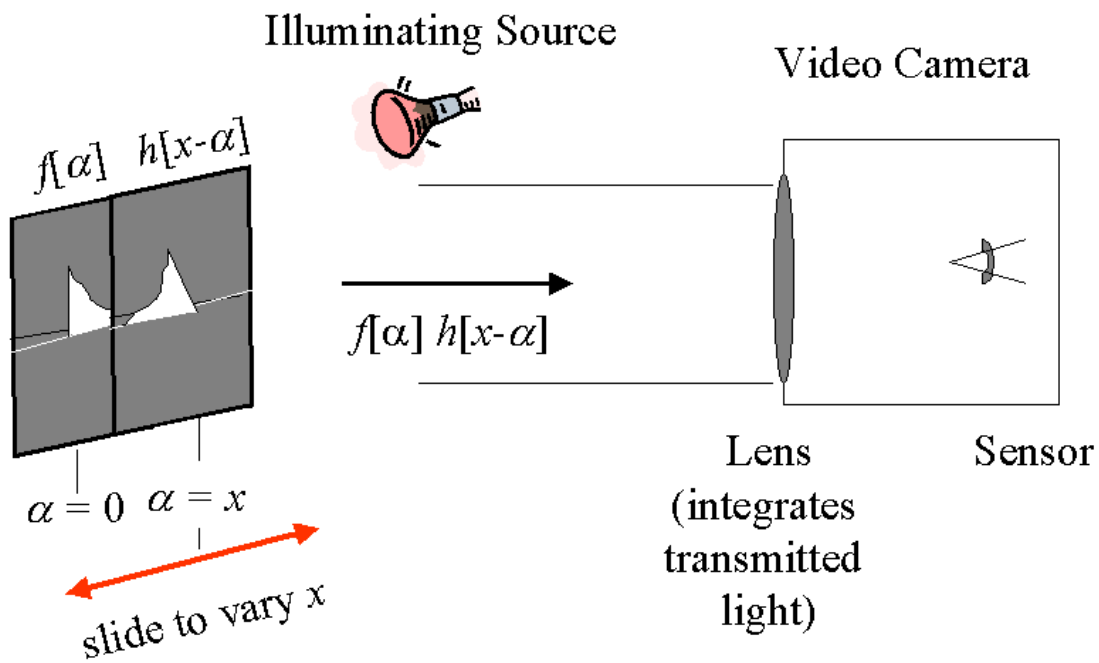
1.2 (1) Optical Implementation of 1-D Graphical Convolution

1.2.1 Required Equipment and Materials

1. illumination source
2. positive lens to collect the light
3. light-sensitive detector and digital oscilloscope OR video camera and digitizer
4. spreadsheet or graphing utility program

1.2.2 Procedure:

Construct the setup:



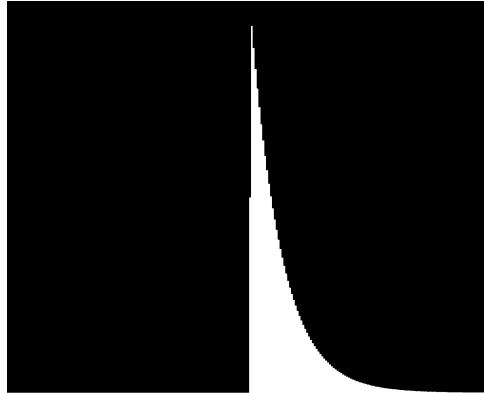
Collect the light from the transparency on the detector and measure the relative intensity with the oscilloscope, or image the scene with a digital camera and integrate the signal digitally to obtain a measurement of the area of the product function.

1.2.3 Experiments:

This setup will be used to convolve $f[x] = e^{-x}STEP[x]$ with other transparencies for the functions:

1. Approximation of Dirac delta function ($h[x] \simeq \delta[x]$)
2. Approximation of even pair of offset Dirac delta functions ($h[x] \simeq \delta[x + x_0] + \delta[x - x_0]$)
3. itself ($h[x] = f[x] = e^{-x}STEP[x]$)
4. with a “reversed” replica ($h[x] = f[-x] = e^{+x}STEP[-x]$)

Create “image” representations of the functions listed below as 2-D transparencies where the area of the function is transparent and the other areas are opaque. The transparency of $e^{-x}STEP[x]$ should look like:



- Make the transparencies small enough to image the entire scene with the video camera or collect all of the light onto the detector.
- A measurement scale along the bottom of the transparency will be convenient
- Be sure to include significant margins on both sides of the origin.
- Make TWO copies of this transparency and single copies of the others.
- Record the transmitted light for several values of x (say 10 or more on either side of the origin), and plot up the recorded numbers vs. x to obtain an graphical approximation of the convolution.

1.2.4 Examples:

Evaluate samples of the convolutions of the following functions and plot them. Compare your results to graphs of the mathematical convolution.

1. $STEP[x] * STEP[x]$
2. $STEP[x] * \delta[x]$
3. $\delta[x] * STEP[x]$
4. $STEP[x] * STEP[-x]$
5. $e^{-x}STEP[x] * \delta[x]$
6. $e^{-x}STEP[x] * (\delta[x + x_0] + \delta[x - x_0])$
7. $e^{-x}STEP[x] * e^{-x}STEP[x]$
8. $e^{-x}STEP[x] * e^xSTEP[-x]$

1.2.5 Questions:

1. What factors affect the accuracy of the optical model of convolution?
2. Propose (and perhaps even implement) a calibration to normalize the evaluated areas.
3. How could you create “bipolar” functions (i.e., with negative amplitudes)?
4. Try to think of a method for evaluating convolutions of complex-valued functions.

1.3 (2) Convolution of Temporal Signals

This experiment performs a similar sequence of operations on temporal signals, e.g., $f [t]$. I suggest not beginning this part until you have finished the first part. In other words, you may postpone this experiment until next week.

1.3.1 Required Equipment and Materials

1. computer with sound card, microphone, and speakers
2. IDL

1.3.2 Procedure:

1. Create a 1-D sound file in the “wav” format using the microphone or other source. You can record your voice, musical notes, or use a tone generator.
2. Use the IDL filtering routines to convolve the sound file with rectangle functions of varying lengths but the same areas.
3. Compare graphical and audio renditions of the original and filtered signals.

1.3.3 Questions:

1. Describe the effect of the filtering on the signal in qualitative terms.
2. Which temporal frequencies (“high” or “low”) are most obviously affected by the convolution?
3. Describe how the system could be changed to attenuate the complementary frequencies (e.g., “low” or “high”).