

# SIMG-717-20052 Homework #6 Due 2/22/2006 (W)

1. The following signals are applied independently to LSI systems:

$$\begin{aligned} s_1[x] &= e^{-x} \cdot STEP[x] \\ s_2[x] &= RECT[2x] * (\delta[x] + \delta[x - 4] + \delta[x - 7] + \delta[x - 9]) \end{aligned}$$

For both cases:

- (a) Describe the impulse response  $h[x]$  and transfer function  $H[\xi]$  of the matched filter that will maximize the output at  $x = 2$ . (assume that  $H[0] = 1$ ). Sketch the output.
  - (b) Is it possible to construct a transfer function  $H[\xi]$  that, when applied to  $s[x]$ , will produce  $g[x] = \delta[x - 2]$ ? Explain your reasoning.
2. The transfer functions listed below describe the action of different LSI systems. The goal of this problem is to find the corresponding “inverse filter” in both domains, i.e.,

$$\begin{aligned} W[\xi] &= (H[\xi])^{-1} \\ w[x] &= \mathcal{F}^{-1}\{W[\xi]\} \end{aligned}$$

In the situations where the inverse filter does not exist, we will instead evaluate the “pseudoinverse” filter

$$\begin{aligned} \hat{W}[\xi] &= \begin{cases} (H[\xi])^{-1} & \text{for } H[\xi] \neq 0 \\ 0 & \text{for } H[\xi] = 0 \end{cases} \\ \hat{w}[x] &= \mathcal{F}^{-1}\{\hat{W}[\xi]\} \end{aligned}$$

In each case, determine which of the inverse filters is appropriate and sketch its transfer function  $W_n[\xi]$  or  $\hat{W}_n[\xi]$ . Classify the action of the appropriate filter as lowpass, highpass, etc. ALSO in those cases where it is possible, evaluate and sketch the appropriate impulse response ( $w_n[x]$  or  $\hat{w}_n[x]$ ) of the appropriate inverse filter. You may use reasonable approximations where appropriate – the sketches may be helpful here.

- (a)  $H_1[\xi] = GAUS[\xi]$
  - (b)  $H_2[\xi] = RECT[2\xi]$
  - (c)  $H_3[\xi] = e^{+i\pi\xi}$
  - (d)  $H_4[\xi] = e^{+i\pi(1-RECT[\xi])}$
3. Design the Wiener or Wiener-Helstrom filter (whichever is appropriate) for the following input signals, impulse responses, and noise power spectra.

- (a)  $f[x] = 2 GAUS[x]$ ,  $h[x] = \delta[x]$ ,  $|N[\xi]|^2 = GAUS[\xi + \xi_0] + GAUS[\xi - \xi_0]$
- (b)  $f[x] = GAUS[\frac{x}{b}] \cdot \exp[+i\pi x^2]$ ,  $h[x] = RECT[x]$ ,  $|N[\xi]|^2 = GAUS[\xi + \xi_0] + GAUS[\xi - \xi_0]$
- (c)  $f[x] = GAUS[\frac{x}{2}] \cdot \cos[10\pi x]$ ,  $h[x] = \delta[x]$ ,  $|N[\xi]|^2 = GAUS[x]$

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4. Consider a system with the input functions  $f_n[x]$  listed below. The functions are translated by an arbitrary and unknown distance  $x_0$ , so that the actual input function is  $f_n[x - x_0]$ . The goal of this problem is to construct the “matched filter” for these inputs, i.e., determine the impulse response  $m_n[x]$  and/or transfer function  $M_n[\xi]$  that will help determine  $x_0$ . In the ideal case, we can construct a filter such that:

$$f_n[x - x_0] * m_n[x] = \delta[x - x_0]$$

For each of the input functions listed below, determine the transfer functions  $M_n[\xi]$  that produce the ideal input (if possible). ALSO, in those cases where it is possible, evaluate and sketch the impulse response  $m_n[x]$  of the matched filter. Again, you may use reasonable approximations where appropriate.

- (a)  $f_1[x] = GAUS[x]$
- (b)  $f_2[x] = RECT[2x]$
- (c)  $f_3[x] = e^{+i\pi x}$
- (d)  $f_4[x] = e^{+i\pi(1-RECT[x])}$