SIMG-717-20052 Homework #5 Due 2/13/2006 (M)

1. For each of the transfer functions, sketch $H_n[\xi]$ and evaluate and sketch the corresponding impulse response $h_n[x]$. Also classify the filters as lowpass, highpass, phase, etc.

   (a) $H_1[\xi] = e^{+i\pi\xi}$
   (b) $H_2[\xi] = e^{+i\pi}$
   (c) $H_3[\xi] = e^{+i\pi(1-RECT[2\xi])}$
   (d) $H_4[\xi] = RECT[\xi] \cdot H_1[\xi] = RECT[\xi] \cdot e^{+i\pi\xi}$
   (e) $H_5[\xi] = RECT[\xi] \cdot H_3[\xi] = RECT[\xi] \cdot e^{+i\pi(1-RECT[2\xi])}$

2. For the function:

$$f[x] = COMB[x] * RECT[2x]$$

   (a) Sketch $F[\xi]$

   For each of the transfer functions listed, sketch $H[\xi]$, the corresponding impulse response $h[x]$, and the output that results if $f[x]$ is applied to the input. Also classify the filters as lowpass, highpass, phase, etc.

   (b) $H[\xi] = RECT[\xi]$
   (c) $h[x] = RECT[x]$
   (d) $H[\xi] = -RECT[4\xi+2] + RECT[4\xi-2]$
   (e) $H[\xi] = 1+\xi^2$

3. Show that the autocorrelation of the impulse response of the constant-phase and linear-phase allpass filters are on-axis Dirac delta functions.

4. Evaluate the *M-C-M* or *C-M-C* chirp Fourier transforms (whichever is more convenient)

$$\begin{aligned} F\left[\frac{x}{\alpha^2}\right] &= \left(\left(f[x] \cdot e^{-i\pi\left(\frac{x}{\alpha}\right)^2}\right) * e^{+i\pi\left(\frac{x}{\alpha}\right)^2}\right) \cdot e^{-i\pi\left(\frac{x}{\alpha}\right)^2} \\ &= \frac{1}{|\alpha|} e^{-i\frac{\pi}{4}} \cdot \left(\left\{\left(f[x] * e^{+i\pi\left(\frac{x}{\alpha}\right)^2}\right) \cdot e^{-i\pi\left(\frac{x}{\alpha}\right)^2}\right\} * e^{+i\pi\left(\frac{x}{\alpha}\right)^2}\right) \end{aligned}$$

   for the following functions

   (a) $f[x] = \delta[x]$
   (b) $f[x] = \delta[x-x_0]$
   (c) $f[x] = 1[x]$
   (d) $f[x] = \exp[+2\pi i\xi_0 x]$
   (e) $f[x] = \exp[+i\pi x^2]$

5. Use the *C-M-C* chirp Fourier transform to find an expression for $f[x] * \exp\left[+i\pi\left(\frac{x}{\alpha}\right)^2\right]$ in terms of $F\left[\frac{x}{\alpha^2}\right]$.