SIMG-717-20052
Homework Assignment #3 Due 1/16/2006 (M)

Midterm Exam: 1/23/3006 (M)

1. A sampling function $s[x;\Delta x]$ is used to sample a signal $f[x]$

$$f_s[x] = f[x] \cdot s[x;\Delta x]$$

. The sampled function $f_s[x]$ is then passed through an LSI system with transfer function $H[\xi]$ to produce an output $g[x]$. For:

$$\begin{aligned} f[x] &= SINC^2[100x] \\ s[x;\Delta x] &= \frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right] \\ H[\xi] &= RECT[\Delta x \cdot \xi] : \end{aligned}$$

   (a) Find the minimum sampling rate $\xi = \xi_{\text{Nyquist}}$ (Nyquist rate) that will permit exact recovery of $f[x]$ from $f_s[x]$; i.e., such that $g[x] = f[x]$ (to within a multiplicative constant).

   (b) Find $g[x]$ when $\xi_s = 0.5\xi_{\text{Nyquist}}$, and sketch.

2. A function $f[x]$ is digitized in a stepwise fashion to produce

$$f_d[x] = (f[x] * p[x]) \cdot s[x;\Delta x]$$

which is then passed through an LSI filter with transfer function $H[\xi]$ to produce an output $g[x]$. For

$$\begin{aligned} f[x] &= SINC[10x] \\ p[x] &= 100 \cdot RECT[100x] \\ s[x;\Delta x] &= RECT\left[\frac{x}{5}\right] \cdot \left(\frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right]\right) \\ H[\xi] &= RECT[\Delta x \cdot \xi] \end{aligned}$$

   (a) Find $g[x]$ if ξ_s is chosen to be the usual Nyquist rate associated with a COMB sampling function of infinite extent.

   (b) Find $g[x]$ if ξ_s is chosen to be twice the Nyquist rate determined in part (a).

   (c) What parameter changes will improve signal recovery?

   (d) Can $f[x]$ ever be recovered exactly in this case?

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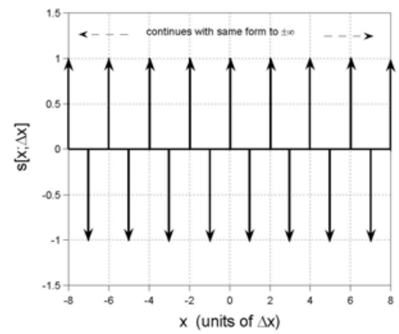
3. Evaluate the following operations:

   (a) $\frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right] * RECT\left[\frac{x}{\Delta x}\right]$
   (b) $\frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right] * TRI\left[\frac{x}{\Delta x}\right]$
   (c) $\frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right] * SINC\left[\frac{x}{\Delta x}\right]$
   (d) $\frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right] * SINC^2\left[\frac{x}{\Delta x}\right]$

4. Analyze the sampling functions shown and specified below to determine their sampling characteristics, e.g., the associated Nyquist sampling frequency (if any), whether a function sampled with $s[x;\Delta x]$ may be recovered from the samples, the corresponding interpolation function, etc.

   (a) $s_1[x;\Delta x] = \left(\frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right]\right) \cdot \left(TRI\left[\frac{x}{b}\right] * \frac{1}{2b} COMB\left[\frac{x}{2b}\right]\right)$; where $b > \Delta x$. Consider any interplay of the factors Δx and b.

   (b) $s_2[x;\Delta x]$ as shown in the figure:



5. A 1-D function $f[x]$ is sampled at N pixels to produce $f_s[x;\Delta x, N]$. The continuous function may be translated by the distance x_0 by convolution with $h[x] = \delta[x - x_0]$. The impulse response of the corresponding discrete operation is $\delta_d[n - n_0]$, where $x_0 = n_0 \cdot \Delta x$, Δx is the sampling interval, and the "discrete delta function" is defined:

$$\delta_d[n - n_0] = \begin{cases} 1 & if \quad n = n_0 \\ \\ 0 & if \quad n \neq n_0 \end{cases}$$

   This formulation for discrete translation "works" only when n_0 is an integer. However, it is often necessary to translate a 1-D discrete function by a noninteger number of samples.

   (a) Derive a rigorous formulation for translating $f[n]$ by a noninteger number of samples if the discrete function $f[n]$ has infinite support. Include sketches of the discrete transfer function and impulse response.

   (b) Adapt the process derived in (a) for the case where $f[n]$ is defined over a finite array of size N samples. Again, include sketches of the transfer function and impulse response, and also discuss the limitations of the process compared to that in (a).