SIMG-717-20052 Midterm Exam 25 January 2006

Two Hours, Select Five (5) (equal weighting, if not equal difficulty)

1. Consider the 2-D function:

$$f[x,y] = CYL\left(\frac{r}{2}\right) * \{\delta[x-1,y-1] + \delta[x+1,y+1]\}$$
$$+ RECT\left[\frac{x}{2},\frac{y}{2}\right]\{\delta[x-1,y+1] + \delta[x+1,y-1]\}$$

where the symbol "$*$" denotes the usual 2-D convolution operation.

   (a) Sketch the "top view" of $f[x,y]$, including labels for amplitudes and coordinates.

   (b) Evaluate $\mathcal{F}_2\{f[x,y]\} \equiv F[\xi,\eta]$. You may use any known theorems or Fourier transforms of special functions without proof but specify what you are using.

   (c) Evaluate the Radon transform $\mathcal{R}_2\{f[x,y]\} \equiv \lambda_f(p,\phi)$ for $\phi = 0$ and $\phi = +\frac{\pi}{2}$ radians. Again, you may use any known Radon transforms without proof but specify what that you are using.

   (d) Evaluate the 1-D Fourier transforms of the two projections at $\phi_1 = 0$ and at $\phi = +\frac{\pi}{2}$ radians and use these results to confirm (not "prove") the central-slice theorem.

2. For each of the following functions:

$$f_1[x] = SINC[2x]$$
$$f_2[x] = SINC^2[2x]$$

   (a) Determine the Nyquist sampling frequencies ξ_{Nyquist}

   (b) For ONLY the single function $f_n[x]$ with the LARGER value of ξ_{Nyquist}, make sketches (with appropriate axis labels) of the following three cases in the space domain:

      1. $f_n[x]$ and the result of ideal sampling at the Nyquist rate
      2. $f_n[x]$ and the result of ideal sampling at half of the Nyquist rate,
      3. $f_n[x]$ and the result of ideal sampling at twice the Nyquist rate.

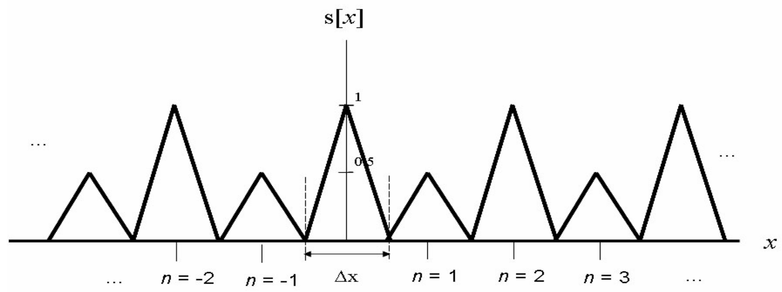
   (c) Evaluate the continuous functions that would be "reconstructed" from the sets of samples after interpolation with the appropriate functions.

   (d) Comment on the results of part c with respect to the Whittaker-Shannon sampling theorem.

3. Evaluate the Radon transform of

$$f[x,y] = \exp\left[+i\pi\frac{x^2+y^2}{a^2}\right]$$

4. A 1-D imaging sensor is composed of an infinite number of detector elements of width Δx. The "response" (sensitivity function) of a small portion of the sensor is shown below. Note that the response function continues with this form to $x = \pm\infty$.



(a) Write down a mathematical expression for the samples generated by this detector from a 1-D input signal $f[x]$. Assume that $f[x]$ is bandlimited so that $\xi_{\max} < \frac{1}{2(\Delta x)}$ (there are several ways to do this).

(b) Evaluate the Fourier transform of this sampled signal. You may assume any form for $F[\xi]$ within the band limit.

(c) Determine if it is *theoretically* possible to recover $f[x]$ from these samples. If not, give reasons. If so, then list the necessary steps in the procedure.

5. Consider $f[x] = TRI[x-2]$

(a) Derive the general expression for the ℓ^{th} moment of $f[x]$.

(b) Write down an approximate expression for $F[\xi]$ that would be valid for $|\xi| \simeq 0$ and confirm it by comparison to $\mathcal{F}\{TRI[x-2]\}$

6. Consider the convolution of an arbitrary input function $f[x,y]$ with the 2-D function $J_0(2\pi r\rho_n)\cdot 1(\theta)$ (where ρ_n is a specific radial spectral frequency indexed by n).

(a) Derive the expression for the spectrum of the output function $g[x,y]$.

(b) If the same filtering process is realized for the same input function $f[x,y]$ but using different values of the spectral frequency ρ_n, describe the differences among the resulting output functions

$$g_n[x,y] = f[x,y] * (J_0(2\pi r\rho_n)\cdot 1(\theta))$$

7. Do FOUR (4) of the following short-answer questions:

(a) Evaluate the volume of $RECT\left(\frac{x}{2},\frac{y}{2}\right) * CYL\left(\frac{r}{2}\right)$.

(b) Determine the regions of support of $e^{-i\pi x^2} * e^{+i\pi x^2}$ and of $e^{+i\pi x^2} * e^{+i\pi x^2}$

(c) Evaluate $TRI\left[2x,\frac{y}{2}\right] * (1[x]\cdot\delta[y])$

(d) Evaluate $(1[x]\cdot\delta[y]) * (\delta[x]\cdot 1[y])$

(e) Find the general expression for the line-integral projections of the 2-D function $\delta[x-1,y-2]$.