1. The impulse response of an imaging system is:

\[
h(x) = \exp \left[ +i\pi \left( \frac{x}{\alpha_0} \right)^2 \right]
\]

Evaluate the expression for the inverse filter \( w[x] \)

2. The transfer function of a filter is:

\[
H_1 [\xi] = 1 [\xi] \cdot \exp [i \cdot 2\pi \cdot (1 - 2 \cdot \text{RECT}[\xi])]
\]

(a) Characterize this filter (highpass, ...)

(b) Evaluate and sketch the impulse response \( h[x] \)

Find and sketch the outputs \( g[x] \) for the following inputs:

(c) \( f_1[x] = \cos \left( \frac{2\pi x}{8} \right) \)

(d) \( f_2[x] = \cos [4\pi x] \)

(e) Repeat (c) and (d) for the modified transfer function:

\[
H_2 [\xi] = 1 [\xi] \cdot \exp \left[ i \cdot 2\pi \cdot \left( \frac{1}{2} - \text{RECT}[\xi] \right) \right]
\]

3. The transfer function of a 1-D system is:

\[
H[\xi] = \left( 1 - 2 \cdot \text{RECT} \left[ \frac{\xi}{0.5} \right] \ast \text{COMB}[\xi] \right) \cdot \exp \left[ +i\pi \left( 1 - \text{COMB}[\xi] \ast \text{RECT}[2\xi] \right) \right]
\]

(a) Sketch this transfer function as (real, imaginary) parts and as (magnitude, phase).

(b) Classify this filter (e.g., highpass, ...).

(c) Derive and sketch the impulse response \( h[x] \).

(d) Derive and sketch the inverse filter \( w[x] \) for this impulse response.

4. Determine the areas of:

(a) \( \cos [2\pi x] \cdot \cos [\pi x^2] \)

(b) \( \text{SINC}^4 [x] \)

5. An imaging system acting on the 1-D input function \( f[x] \) includes the following steps:

(a) evaluate \( F \{ f[x] \} |_{\xi=x/(\alpha_0)^2} \) where \( \alpha_0 \) is a constant parameter with units of “length”;

(b) multiply by the real-valued pupil function \( p[x] \) (which has finite support) to form \( g_1[x] \)

(c) evaluate \( F \{ g_1[x] \} |_{\xi=x/\alpha_0} \)

For an input function of your choice, evaluate and sketch the output function if the pupil \( p[x] = \text{RECT} \left[ \frac{x}{\alpha_0} \right] \). Note that this is a real imaging system where the first and last steps propagate light a long distance to the Fraunhofer diffraction region.
6. Modify the imaging system in the previous problem so that the explicit steps in the process are now:

(a) evaluate $\mathcal{F}\{f[x]\}|_{x=\xi}$, where $\alpha_0$ is a constant parameter with units of “length”;

(b) multiply by $p[x] = \text{RECT} \left[ \frac{x}{b_0} \right] \cdot \exp \left[ -i\pi \left( \frac{x}{\alpha_1} \right)^2 \right]$, where both $b_0$ and $\alpha_1$ are system parameters with units of length

(c) convolve the product in step (b) with impulse response $h[x] = \exp \left[ +i\pi \left( \frac{x}{\alpha_1} \right)^2 \right]

In this system, steps (b) and (c) are the first two steps in an $M$-C-M chirp Fourier transformer. This is also a realistic imaging imaging system step (a) represents propagation from the object to a location in the Fraunhofer diffraction region, step (b) is the action of a lens, and step (c) is propagation to the focal plane of the lens. In this example, the constants $\alpha_1 = \sqrt{\lambda_0 z_1}$ and $\alpha_1 = \sqrt{\lambda_0 f}$, where $z_1$ is the propagation distance from the object to the lens and $f$ is the focal length of the lens.

7. A measured signal $g[x]$ is the sum of a deterministic part $f[x]$ and a “stochastic” random part $n[x]$:

(a) Describe the constraints on $f[x]$ AND on $n[x]$ that would allow a filter $w[x]$ to be constructed such that the output of the filter is exactly equal to $f[x]$, i.e.,

$$(f[x] + n[x]) * w[x] = f[x]$$

(b) Specify AND SKETCH the transfer function AND impulse response of the filter that will “extract” $f[x]$ from $g[x]$ if the signal and noise spectra are the functions:

$$F[\xi] = \text{STEP} [\xi] \cdot \exp \left[ +i\pi \xi^2 \right]$$

$$N[\xi] = \text{STEP} [-\xi] \cdot \exp \left[ +i\pi \cdot R[\xi] \right]$$

where $R[\xi]$ is a random number selected from the interval $-\pi \leq R[\xi] < +\pi$

8. We already know that the output of a “perfect” imaging system presented with the input $f[x]$ is $f[x]$: $f[x] * \delta[x] = f[x]$

The output of a second imaging system consists of the sum of two identical but translated replicas of the input, where the two replicas are displaced by the distance $b_0$.

(a) Write down the expression for the impulse response of the second system.

(b) Evaluate and sketch the outputs for the following inputs:

$$f_1[x] = \cos \left[ 2\pi \left( \frac{x}{b_0} + \frac{1}{2} \right) \right]$$

$$f_2[x] = \cos \left[ 2\pi \left( \frac{x}{2b_0} \right) \right]$$

(c) Evaluate and sketch the transfer function of the system.

(d) Is it possible to extract the input $f[x]$ from the output $g[x]$ and the impulse response $h[x]$? Explain why or why not.

9. Modify the previous problem so that the second exposure of the system is attenuated relative to the first by a factor of 2.

10. Modify the problem so that the image is not created from the two exposures but rather from a single exposure made over the displacement by the distance $b_0$. 

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