1051-716-20111 Homework Assignment #8 Due 11/9/2011 (W)

0. Read §18 *Magnitude-Phase Filters* and §19.*Applications of Linear Filters*

1. A signal $s[x] = \exp[-x] \cdot STEP[x]$ is the input to an LSI system.

   (a) Find and sketch the output $g[x]$ if $h[x] = s[x]$

   (b) Find and sketch the output $g[x]$ if $h[x] = s[-x]$

   (c) Describe the impulse response $h[x]$ and transfer function $H[\xi]$ of the matched filter that will maximize the output at $x = x_0$, i.e., at some arbitrary location.

   (d) Find and sketch the output if $h[x] = s[-x]$ and the input is

$$s[x] * \left(\frac{1}{2}\delta[x-2] + \frac{1}{3}\delta[x+3]\right)$$

2. Repeat the four parts of #1 if $s[x] = \exp\left[+i\pi\frac{x^2}{2}\right]$

3. The transfer functions listed below describe the actions of different LSI systems. The goal of this problem is to find the corresponding inverse filter. In each case, sketch the transfer function of the inverse filter. ALSO *in those cases where it is possible*, evaluate and sketch the impulse response of the inverse filter. You may use reasonable approximations where appropriate – the sketches will be helpful here.

   (a) $H[\xi] = GAUS[\xi]$

   (b) $H[\xi] = \exp[+2\pi i\xi]$

   (c) $H[\xi] = \exp[+i\pi(1 - RECT[\xi])]$

   (d) $H[\xi] = \exp\left[+i\pi\frac{\xi^2}{2}\right]$

4. A desired signal $f[x]$ is corrupted by additive noise $n[x]$, and you are required to design a filter that can be used to recover the signal for the following cases. Find a transfer function $H[\xi]$ such that the output $g[x] \cong s[x]$ when the input is $f[x] = s[x] + n[x]$. Sketch $f[x]$ and $g[x]$.

   (a)

$$s[x] = \left(\frac{1}{5}COMB\left[\frac{x}{5}\right] * RECT[x]\right)\cos[60\pi x]$$
$$n[x] = \left(\frac{1}{2}COMB\left[\frac{x}{2}\right] * TRI[x]\right)\cos[20\pi x]$$

   (b)

$$s[x] = GAUS\left[\frac{x}{5}\right]$$
$$n[x] = GAUS\left[\frac{x}{10}\right] \cdot \cos[\pi x]$$

5. The input to an LSI system is:

$$s[x] = RECT[2x] * (\delta[x] + \delta[x-4] + \delta[x-7] + \delta[x-9])$$

   (a) Find and sketch the output $g[x]$ when $h[x] = s[x]$

   (b) Find and sketch the output $g[x]$ when $h[x] = s[-x]$

   (c) Describe the impulse response $h[x]$ and transfer function $H[\xi]$ (assume that $H[0] = 1$) of the matched filter that will maximize the output at $x = 2$. Sketch the output.

(d) For this $s[x]$, does the transfer function $H[\xi]$ exist such that $g[x] = s[x] * h[x] = \delta[x-2]$? Explain your answer.

6. Design the Wiener-Helstrom filter for the following input signals, impulse responses, and noise power spectra.

   (a) $f[x] = RECT[x]$, $h[x] = RECT[x]$, $|N[\xi]|^2 = RECT[\xi+1] + RECT[\xi-1]$

   (b) $f[x] = 2\,GAUS[x]$, $h[x] = \delta[x]$, $|N[\xi]|^2 = GAUS[\xi+\xi_0] + GAUS[\xi-\xi_0]$

7. A 1-D image $g[x]$ has been created by a double exposure of the original object $f[x]$. The original scene was translated between the exposures by the known distance $+b$. The object was stationary during the time that each image was collected, and the exposure time was the same in both cases.

   (a) Design the inverse filter for this system in the frequency domain. Comment about the potential of success of the deblurring process, particularly if noise is present.

   (b) Find an exact or approximate expression for the inverse filter in the space domain.