1051-716-20091 Homework Assignment #8 Due 11/16/2009 (M)

(You may hand in to me or slide under my door)

0. Read §18 *Magnitude-Phase Filters* and §19.*Applications of Linear Filters*

1. A signal $s[x] = e^{-x} \cdot STEP[x]$ is the input to an LSI system.

   (a) Find and sketch the output $g[x]$ if $h[x] = s[x]$

   (b) Find and sketch the output $g[x]$ if $h[x] = s[-x]$

   (c) Describe the impulse response $h[x]$ and transfer function $H[\xi]$ of the matched filter that will maximize the output at $x = x_0$, i.e., at some arbitrary location.

   (d) Find and sketch the output if $h[x] = s[-x]$ and the input is

$$s[x] * \left(\frac{1}{2}\delta[x-2] + \frac{1}{3}\delta[x+3]\right)$$

2. Repeat the four parts of #1 if $s[x] = \exp\left[+i\pi\frac{x^2}{2}\right]$

3. The transfer functions listed below describe the actions of different LSI systems. The goal of this problem is to find the corresponding inverse filter. In each case, sketch the transfer function of the inverse filter. ALSO *in those cases where it is possible*, evaluate and sketch the impulse response of the inverse filter. You may use reasonable approximations where appropriate – the sketches will be helpful here.

   (a) $H[\xi] = GAUS[\xi]$

   (b) $H[\xi] = \exp[+2\pi i\xi]$

   (c) $H[\xi] = \exp[+i\pi(1 - RECT[\xi])]$

   (d) $H[\xi] = \exp\left[+i\pi\frac{\xi^2}{2}\right]$

4. A desired signal $f[x]$ is corrupted by additive noise $n[x]$, and you are required to design a filter that can be used to recover the signal for the following cases. Find a transfer function $H[\xi]$ such that the output $g[x] \cong s[x]$ when the input is $f[x] = s[x] + n[x]$. Sketch $f[x]$ and $g[x]$.

   (a)

$$\begin{aligned} s[x] &= \left(\frac{1}{5}COMB\left[\frac{x}{5}\right] * RECT[x]\right)\cos[60\pi x] \\ n[x] &= \left(\frac{1}{2}COMB\left[\frac{x}{2}\right] * TRI[x]\right)\cos[20\pi x] \end{aligned}$$

   (b)

$$\begin{aligned} s[x] &= GAUS\left[\frac{x}{5}\right] \\ n[x] &= GAUS\left[\frac{x}{10}\right] \cdot \cos[\pi x] \end{aligned}$$

5. A 1-D image $g[x]$ has been created by a time exposure of an object $f[x]$ that is moving past the imaging system at a constant speed during the exposure. Over the time T_0 of the exposure, the object is translated by the known distance $+b_0$ on the sensor.

   (a) Design the inverse filter for this system in the frequency domain.

   (b) Comment about the potential of success of the deblurring process, particularly if noise is present.

6. Design the Wiener or Wiener-Helstrom filter for the following input signals, impulse responses, and noise power spectra. Again (and as usual), sketches of the transfer functions may be helpful.

   (a) $f[x] = 2\ GAUS[x]$, $h[x] = \delta[x]$, $|N[\xi]|^2 = GAUS[\xi + \xi_0] + GAUS[\xi - \xi_0]$

   (b) $f[x] = GAUS\left[\frac{x}{2}\right] \cdot \cos[10\pi x]$, $h[x] = \delta[x]$, $|N[\xi]|^2 = GAUS[x]$