1051-716-20111 Homework #7 Due 10/31/2011 (M)

0. Read §16 (Magnitude Filtering) and §17 (Phase Filtering)

1. For the input function:
$$f[x] = \frac{1}{2} COMB\left[\frac{x}{2}\right] * RECT[x]$$
   (a) Find an expression for and sketch $F[\xi]$
   (b) Find an expression for a transfer function $H[\xi]$ that is an bandpass filter that passes only the spatial frequencies with orders ± 1;sketch the output.
   (c) Find an expression for a transfer function $H[\xi]$ that is an bandpass filter that passes only the spatial frequencies with order $+1$;sketch the output.

2. For
$$f[x] = A_0 + A_1 \cos[2\pi\xi_0 x + \phi_0]$$
   where $A_0 \geq A_1$ and ξ_0 and ϕ_0 are real-valued parameters.
   (a) Sketch $f[x]$
   (b) Sketch $F[\xi]$
   (c) Find an expression for the modulation of $f[x]$.

3. The input in the previous problem is applied to various LSI systems described by the following transfer functions. In each case, characterize the action of the filter (e.g., highpass, lowpass, etc.). For each, sketch the transfer function $H[\xi]$, the corresponding impulse response $h[x]$, the output spectrum $G[\xi]$, and the output $g[x]$. If the modulation of the output is sensibly defined, specify it. (Most of these should take very little time)
   (a) $h[x] = RECT\left[\frac{x}{b_0}\right]$, where $b_0 = (2\xi_0)^{-1}$
   (b) $h[x] = RECT\left[\frac{x}{b_0}\right]$, where $b_0 = \xi_0^{-1}$
   (c) $h[x] = RECT\left[\frac{x}{b_0}\right]$, where $b_0 = 2 \cdot \xi_0^{-1}$
   (d) $h[x] = \frac{1}{b_0} \cdot SINC\left[\frac{x}{b_0}\right]$
   (e) $h[x] = \frac{1}{2} \cdot SINC\left[\frac{x}{2}\right] \cdot \exp[+2\pi i\xi_0 x]$
   (f) $H[\xi] = \exp[+i\pi]$
   (g) $H[\xi] = \exp\left[+i\pi \cdot RECT\left[\frac{\xi}{\xi_0}\right]\right]$
   (h) $H[\xi] = \exp\left[+i\pi\left(1 - RECT\left[\frac{\xi}{\xi_0}\right]\right)\right]$
   (i) $H[\xi] = \exp\left[+i\pi\left(1 - TRI\left[\frac{\xi}{4 \cdot \xi_0}\right]\right)\right]$

4. Given a linear shift-invariant system with inputs $f_n[x]$, outputs $g_n[x]$, and the single impulse response $h[x] = 7 \cdot SINC[7x]$. Find the outputs of the following by evaluating the product of the spectra and the subsequent inverse Fourier transform.
   (a) $f_1[x] = \cos[4\pi x]$
   (b) $f_2[x] = \cos[4\pi x] \cdot RECT\left[\frac{x}{75}\right]$
   (c) $f_3[x] = (1 + \cos[8\pi x]) \cdot RECT\left[\frac{x}{75}\right]$

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5. Given an LSI system and a rectangle-wave input of finite extent described by:

$$f[x] = \left(\frac{1}{2} COMB\left[\frac{x}{2}\right]\ RECT\left[\frac{x}{50}\right]\right) * RECT[x]$$

Using reasonable approximations where appropriate, find the output for each of the following transfer functions. Sketch the transfer function, impulse response, output spectrum, and output in each case

(a) $H[\xi] = \exp[+i\pi]$

(b) $H[\xi] = \exp[+i\pi\xi]$

(c) $H[\xi] = \exp[+i\pi \cdot RECT[2\xi]]$