1051-716-20091 Homework Assignment #7 Due 11/4/2009 (W)

0. Finish skimming §12 of notes *Radon Transform,* read §16 *Magnitude Filtering* and §17.*Phase Filtering* (EXCEPT for §17.5 on *Higher-Order Phase*)

1. Find expressions for and sketch the transfer functions of imaging systems with the following impulse responses:

   (a) $h(r) = J_0(2\pi r) + J_0(\pi r)$

   (b) $h(r) = SOMB\left(\frac{r}{10}\right)$

2. Evaluate AND sketch the results of the following operations, where the symbols "$*$" and "$\bigstar$" denote 2-D convolution and correlation, respectively: Note that it may be very helpful to sketch the spectra of the components.

   (a) $CYL\left(\frac{r}{2}\right) * (\delta[x] \cdot 1[y])$

   (b) $J_0(2\pi\rho_0 r) * J_0(2\pi\rho_1 r)$, where $\rho_0 \neq \rho_1$

   (c) $(\cos[\pi\xi] \cdot SINC[\eta] + SINC[\xi] \cdot \cos[\pi\eta]) \bigstar \left(\cos[2\pi\xi] \cdot SINC\left[\frac{\eta}{2}\right] + SINC\left[\frac{\xi}{2}\right] \cdot \cos[2\pi\eta]\right)$

3. Given an LSI system and an input whose magnitude is constant but whose phase is a square wave, i.e.,:

$$\begin{aligned} f[x] &= 1[x] \cdot \exp[+i\Phi[x]] \\ \Phi[x] &= -\pi + \pi \cdot \left(\left(COMB[x] \cdot RECT\left[\frac{x}{101}\right]\right) * RECT[4x]\right) \end{aligned}$$

   (a) Sketch the phase function $\Phi[x]$ and the input function $f[x]$.

   (b) Show that the input may be written as:

$$f[x] = \left(2 \cdot \left(COMB[x] \cdot RECT\left[\frac{x}{101}\right]\right) * RECT[4x]\right) - 1[x]$$

   (c) For the transfer function:

$$H[\xi] = RECT\left[\frac{\xi}{51}\right] \cdot \exp[+i\pi \cdot RECT[8\xi]]$$

   show that the output is approximately equal to:

$$g[x] \cong 2 \cdot \left(COMB[x] \cdot RECT\left[\frac{x}{101}\right]\right) * RECT[4x]$$

   (d) Sketch $g[x]$

4. For the input function:

$$f[x] = \frac{1}{2} COMB\left[\frac{x}{2}\right] * RECT[x]$$

   (a) Find an expression for and sketch $F[\xi]$

   (b) Find an expression for a transfer function $H[\xi]$ that is an bandpass filter that passes only the spatial frequencies with orders ± 1 (i.e., the smallest spatial frequencies with nonzero amplitude); sketch the output.

   (c) Find an expression for a transfer function $H[\xi]$ that is an bandpass filter that passes only the spatial frequencies with order $+1$; sketch the output.

MORE→→→

5. For

$$f[x] = A_0 + A_1 \cos[2\pi\xi_0 x + \phi_0]$$

where $A_0 \geq A_1$ and ξ_0 and ϕ_0 are real-valued parameters.

(a) Sketch $f[x]$

(b) Sketch $F[\xi]$

(c) Find an expression for the modulation of $f[x]$.

6. The input in problem 5 is applied to various LSI systems with each of the following transfer functions. In each case, characterize the action of the filter (e.g., highpass) For each, sketch the transfer function $H[\xi]$, the corresponding impulse response $h[x]$, the output spectrum $G[\xi]$, and the output $g[x]$. If the modulation of the output is sensibly defined, specify it. (Most of these should take very little time)

(a) $h[x] = RECT\left[\frac{x}{b_0}\right]$, where $b_0 = (2\xi_0)^{-1}$

(b) $h[x] = RECT\left[\frac{x}{b_0}\right]$, where $b_0 = \xi_0^{-1}$

(c) $h[x] = RECT\left[\frac{x}{b_0}\right]$, where $b_0 = 2 \cdot \xi_0^{-1}$

(d) $h[x] = \frac{|\xi_0|}{2} SINC\left[\frac{x\xi_0}{2}\right]$

(e) $h[x] = \frac{|\xi_0|}{2} SINC\left[\frac{x|\xi_0|}{2}\right] \cdot \exp[+2\pi i \xi_0 x]$

(f) $H[\xi] = \exp[-i\pi]$

(g) $H[\xi] = \exp\left[-i\pi \cdot RECT\left[\frac{\xi}{\xi_0}\right]\right]$

(h) $H[\xi] = \exp\left[-i\pi\left(1 - RECT\left[\frac{\xi}{\xi_0}\right]\right)\right]$

(i) $H[\xi] = \exp\left[-i\pi\left(1 - TRI\left[\frac{\xi}{4\xi_0}\right]\right)\right]$