1051-716-20091 Homework Assignment #6 Due 10/28/2009 (W)

0. Read Chapter 10 of notes *2-D Fourier transforms* and Chapter 11 *Spectra of Circular Functions*, SKIM Chapter 12 *The Radon Transform.* If you have yet more time, start on Chapter 16 *Magnitude Filtering.*

1. Consider the nonlinear operator $\mathcal{N}$ such that

$$\mathcal{N}\{f[x]\} \equiv (1 - \exp[-8 \cdot f[x]]) \cdot RECT\left[f[x] - \frac{1}{2}\right]$$

In words, the amplitude f of the input function is constrained to the interval $0 \leq f \leq 1$.

(a) Graph the "lookup table" $g[f]$ for this nonlinear function.

(b) Find any values of f in the specified interval such that $g = f$

(c) Evaluate and sketch $g[x]$ for $f[x] = \frac{1}{2} + \frac{1}{2}\cos[2\pi x]$ over one cycle of $f[x]$

(d) Find an expression for the Fourier transform of $g[x]$ for the input function $f[x]$ in (c); you may make reasonable approximations as necessary.

(e) Use the result of (d) to construct an analogue of the transfer function for a nonlinear system.

(f) Determine which spatial frequencies (if any) are created out of "thin air."

2. For the 2-D input function:

$$f[x,y] = \cos\left[\frac{\pi}{2}x\right] \cdot 1[y]$$

Find the outputs produced by convolution with the following 2-D impulse responses, i.e., evaluate $g_1[x,y] = f[x,y] * h_1[x,y]$, $g_2[x,y] = f[x,y] * h_2[x,y]$, etc.

(a) $h_1[x,y] = SINC[x,y] = SINC[x] \cdot SINC[y]$

(b) $h_2[x,y] = RECT[x,y] = RECT[x] \cdot RECT[y]$

(c) $h_3[x,y] = SINC\left[\frac{x+y}{\sqrt{2}}, \frac{-x+y}{\sqrt{2}}\right] = SINC\left[\frac{x+y}{\sqrt{2}}\right] \cdot SINC\left[\frac{-x+y}{\sqrt{2}}\right]$

3. Given two LSI systems with impulse responses:

$$\begin{aligned} h_1[x,y] &= 25\ SINC^2[5x,5y] \\ h_2[x,y] &= 25\ SINC^2[5x,5y] \cdot \exp[i\pi x] \end{aligned}$$

find the outputs $g_i[x,y]$ from each system AND for the cascade of the two systems (i.e., $g[x,y] = (f[x,y] * h_1[x,y]) * h_2[x,y])$ for the input signals:

(a) $f_1[x,y] = SINC[x,y]$

(b) $f_2[x,y] = 1 + \cos[4\pi x] + \cos[8\pi x]$

(c) $f_3[x,y] = \delta[x] \cdot 1[y]$ (note that $g_3[x,y]$ is a "line response")

4. Evaluate the following 2-D convolutions and make appropriate sketches of the results:

(a) $\left(\cos\left[2\pi\frac{x}{4}\right] \cdot 1[y]\right) * (\delta[x] \cdot 1[y])$

(b) $\left(\cos\left[2\pi\frac{x}{4}\right] \cdot 1[y]\right) * CROSS[x,y]$ (where $CROSS[x,y] \equiv \delta[x]\ 1[y] + 1[x]\ \delta[y]$)

(c) $e^{+i\pi r^2} * (\delta[x] \cdot 1[y])$, where $r = \sqrt{x^2+y^2}$

(d) $e^{+i\pi r^2} * CROSS[x,y]$

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5. Find the zero-order Hankel transforms of the following functions and make "appropriate" sketches of your results IN BOTH DOMAINS (e.g., axial profiles, top views, perspective plots, etc.)

   (a) $f(r) = CYL(2r)$

   (b) $g(r) = CYL(2r) \bigstar CYL(2r)$ (where "$\bigstar$" denotes 2-D correlation)

   (c) $h(r) = CYL(0.25r) - CYL(0.5r)$

   (d) $t(r) = SOMB^2(r) \bigstar SOMB(5r)$

   (e) $v(r) = -4\pi^2 r^2 \cdot GAUS(2r) = (2\pi i r)^2 \cdot GAUS(2r)$