1051-716-20091 Homework Assignment #5 Due 10/21/2009 (W)

0. Finish reading §9

1. Use the filter theorem to evaluate:

   (a) $GAUS\left[\frac{x}{3}\right] * GAUS\left[\frac{x}{4}\right] = \int_{-\infty}^{+\infty} \exp\left[-\pi\left(\frac{\alpha}{3}\right)^2\right] \exp\left[-\pi\left(\frac{x-\alpha}{4}\right)^2\right] d\alpha$

   (b) $SINC[3x] * SINC[2x] = \int_{-\infty}^{+\infty} \left(\frac{\sin[3\pi\alpha]}{3\pi\alpha}\right)\left(\frac{\sin[2\pi(x-\alpha)]}{2\pi(x-\alpha)}\right) d\alpha$

2. Use the Fourier transform of $e^{+i\pi x^2}$ to derive the 2-D transform $\mathcal{F}_2\{f[x,y]\} = \mathcal{F}_2\left\{e^{+i\pi(x^2+y^2)}\right\} = \mathcal{F}_2\left\{e^{+i\pi r^2}\right\}$; sketch the profiles along the x-axis of the real part, imaginary part, magnitude and phase

3. Use the general form of the derivative theorem in Eq.(9.143) to derive an expression for the "half-order" derivative of $f[x]$, i.e., $\left(\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\right) f[x]$.

4. Use Rayleigh's theorem to evaluate the areas of the following:

   (a) $SINC[x] \cdot e^{-\pi x^2}$
   (b) $SINC^3[x]$
   (c) $SINC^4[x]$
   (d) $SINC^5[x]$

5. Evaluate and sketch $f[x] = SINC[x] * \frac{1}{x}$.

6. Evaluate and sketch the following:

   (a) $\mathcal{F}_1\left\{\cos\left(2\pi\frac{x}{2}\right) + \sin\left(2\pi\frac{x}{2}\right)\right\}$
   (b) $\mathcal{F}_1\left\{\sin\left[\left(\frac{x}{2}\right)^2\right]\right\}$
   (c) $\mathcal{F}_1\left\{SINC\left[\frac{x}{3} - \frac{1}{2}\right]\right\}$

7. Use the theorems of the Fourier transform to prove that:

$$r[x-x_0] \bigstar r[x-x_0] = r[x] \bigstar r[x]$$

8. Evaluate the following Fourier transforms and sketch them as real-and-imaginary parts and as magnitude-phase:

   (a) $SINC[x] \cdot SINC\left[\frac{x}{2}\right]$
   (b) $\cos[\pi x] \cdot GAUS[x]$
   (c) $\cos[2\pi\xi_0 x] \cdot RECT\left[\frac{x}{b_0}\right]$
   (d) $\sin[2\pi\xi_0 x] \cdot RECT\left[\frac{x}{b_0}\right]$