1051-716-20091 Homework Assignment #4 Due 10/12/2009 (M)
Midterm Exam 14 October 2009 (W)

0. Read §8 on Operations, Start to read §9 through §9.7 on Fourier transforms

1. Perform the following convolutions and sketch the result in each case:
   (a) $STEP[x] * STEP[x]$
   (b) $STEP[x] * (x \cdot STEP[x])$
   (c) $(x \cdot STEP[x]) * (x \cdot STEP[x])$
   (d) $STEP[x] * RECT[x]$
   (e) $STEP[x] * TRI[x]$
   (f) $s[x] = STEP[x] * (RECT[x+2] - RECT[x-2])$

2. For $f[x] = e^{-x} \cdot STEP[x]$, evaluate and sketch the following convolutions:
   (a) $f[x] * f[x]$
   (b) $f[x] * f[-x]$

3. You may substitute any function $f[x]$ and any impulse response $h[x]$ in the following expression. State the conditions on $f[x]$ and $h[x]$ that ensure that the result is correct. Explain your reasoning.
$$f[x] * h[x] = f[-x]$$

4. Perform the following convolutions and sketch the result in each case:
   (a) $RECT[x] * RECT[x]$
   (b) $RECT[x] * \delta[x-1]$
   (c) $RECT[x] * RECT[x-1]$
   (d) $RECT[x] * \left(\frac{d}{dx}\delta[x]\right)$
   (e) $RECT[x] * TRI[x]$

5. Show that the following expression is true for arbitrary real values d_0 and b_0:
$$RECT\left[\frac{x}{d_0}\right] * RECT\left[\frac{x}{b_0}\right] = \left(\frac{d_0+b_0}{2}\right) \cdot TRI\left[\frac{x}{\left(\frac{d_0+b_0}{2}\right)}\right] - \left(\frac{|d_0-b_0|}{2}\right) \cdot TRI\left[\frac{x}{\left(\frac{|d_0-b_0|}{2}\right)}\right]$$

6. Evaluate the following 2-D convolutions and produce "appropriate" sketches (i.e., axial profiles, "top views", or perspective views).
   (a) $(RECT[x]\ \ \delta[y]) * (\delta[x]\ \ RECT[y])$
   (b) $COR[x,y] * COR[x,y]$

7. The convolution of two scaled Gaussian functions and of two *SINC* functions are:
   (a) $GAUS\left[\frac{x}{3}\right] * GAUS\left[\frac{x}{4}\right] = \int\limits_{-\infty}^{+\infty} \exp\left[-\pi\left(\frac{\alpha}{3}\right)^2\right] \exp\left[-\pi\left(\frac{x-\alpha}{4}\right)^2\right]\, d\alpha$
   (b) $SINC[3x] * SINC[2x] = \int\limits_{-\infty}^{+\infty} \left(\frac{\sin[3\pi\alpha]}{3\pi\alpha}\right)\left(\frac{\sin[2\pi(x-\alpha)]}{2\pi(x-\alpha)}\right) d\alpha$

   Evaluate these integrals either rigorously (by direct integration) or approximately by graphical means (HINT: the rigorous solution is possible using complex integration and thus requires background that most of you probably do not have. A graphical approximation is reasonable.).
   (c) Comment on the difficulty of the evaluation.
   We will revisit these problems after proving the so-called "filter theorem" of the 1-D Fourier transform, where the solution becomes "trivial".