1051-716-20091 Homework Assignment #3 Due 10/7/2009 (W)

0. Finish §7 in the notes on 2-D Special Functions and §8 on Operations
1. Find the spatial period of the 2-D sinusoidal function $.f[x,y] = \cos\left(2\pi\left(\frac{x}{5}+\frac{y}{12}\right)+\frac{\pi}{6}\right)$.
2. Gaskill describes 2-D Dirac delta functions of the form $\delta[ax+by]$, while I refer to 2-D line delta functions of the form $\delta\left[\underline{\mathbf{r}}\bullet\widehat{\underline{\mathbf{p}}}\right] 1\left[\underline{\mathbf{r}}\bullet\widehat{\underline{\mathbf{p}}}^{\perp}\right] = \delta[x\cos\theta + y\sin\theta]\cdot 1[-x\sin\theta + y\cos\theta]$. Sketch the following 2-D delta functions written in Gaskill's formulation and find the corresponding expressions in the other form. Label all important points on the axes of your sketch.
   (a) $f[x,y] = \delta[3x+4y]$
   (b) $g[x,y] = \delta[3x-4y-5]$
3. Graph axial profiles (i.e., $f[x,0]$ and $f[0,y]$for the following functions:
   (a) $f[x,y] = SINC\left[\frac{x}{2},y\right]$
   (b) $h[x,y] = SOMB\left(\sqrt{x^2+\left(\frac{y}{2}\right)^2}\right)$ (where $SOMB(r) \equiv \frac{2J_1(\pi r)}{\pi r}$)
   (c) $p[x,y] = CYL\left(\frac{\sqrt{x^2+y^2}}{2}\right) - CYL\left(\sqrt{x^2+y^2}\right)$
4. Derive expressions for and sketch (or plot) the even and odd parts of the 2-D function:
$$f[x,y] = CYL\left(\frac{\sqrt{(x+3)^2+y^2}}{2}\right) + RECT[x-1,y-1]$$
5. Evaluate the volumes of and sketch the following functions as "top views". Also find expressions for and sketch the even and odd parts.
   (a) $f[x,y] = RECT\left[\frac{x-2}{2},\frac{y-2}{2}\right] - RECT\left[\frac{x+2}{2},\frac{y-2}{2}\right] - RECT\left[\frac{x-2}{2},\frac{y+2}{2}\right] + RECT\left[\frac{x+2}{2},\frac{y+2}{2}\right]$
   (b) $g[x,y] = CYL\left(\frac{3\sqrt{(x-2)^2+y^2}}{2}\right)+CYL\left(\frac{3\sqrt{(x+2)^2+y^2}}{2}\right)+CYL\left(\frac{3\sqrt{x^2+(y-2)^2}}{2}\right)+CYL\left(\frac{3\sqrt{x^2+(y+2)^2}}{2}\right)$
   (c) $h[x,y] = CYL\left(\frac{\sqrt{x^2+y^2}}{4}\right) - CYL\left(\frac{\sqrt{(x-1)^2+y^2}}{2}\right)$
   (d) $p[x,y] = COMB\left[\frac{x-0.5}{2}\right] RECT\left[\frac{y-\frac{1}{2}}{2}\right]$
   (e) $q[x,y] = \left[CYL\left(\frac{r}{2}\right) - CYL(r)\right]\ STEP[y]$
6. For the 2-D circularly symmetric functions
$$\begin{aligned} f_1(r) &= \frac{\delta(r)}{\pi r} \\ f_2(r) &= \delta\left(r-\frac{1}{2}\right) \end{aligned}$$
   (a) Sketch the functions.
   (b) Calculate their volumes.
7. The 1-D convolution is defined:
$$f[x] * h[x] \equiv \int_{\alpha=-\infty}^{\alpha=+\infty} f[\alpha]\ \ h[x-\alpha]\ \ d\alpha = g[x]$$
   The system denoted by $\mathcal{O}_2$ exhibits the following behaviors:
$$\begin{aligned} \mathcal{O}_2\{f_2[x]\} &= \delta[2x] \\ \mathcal{O}_2\{2f_2[x]\} &= \delta[x] \end{aligned}$$
   Determine (if possible) if the system is linear and/or shift invariant and explain how you know.