1051-716-20091 Homework Assignment #2 Due 9/28/2009 (M)

0. Read §6.1 - §6.3 in the notes (on 1-D special functions) and §7.1 - §7.3 (on 2-D special functions); you may also want to skim §6.4 (on 1-D stochastic functions).

1. Given that $f[x] \equiv RECT[x+2] + RECT[x-2]$, sketch the following functions:

   (a) $f[x]$

   (b) $g[x] \equiv f[x-1]$

   (c) $h[x] \equiv f[x]\ SGN[x]$

   (d) $p[x] \equiv h[x-1]$

   (e) $q[x] = -f[x] \cdot SGN[-x]$

2. Derive an expression for and sketch (or plot) the even and odd parts of:

$$e^{-(x+x_0)} STEP[x+x_0]$$

   where x_0 is a positive real number.

3. Given *positive* real-valued parameters b_0 and x_0, and the function $f[x] \equiv TRI[x] \cdot STEP[x]$, sketch the following:

   (a) $f[x]$

   (b) $f\left[\frac{x}{b_0}\right]$

   (c) $f[x+x_0]$

   (d) $f[-x] = f\left[\frac{x}{-1}\right]$

   (e) $f\left[\frac{x+x_0}{b_0}\right]$

   (f) $f\left[\frac{-x-x_0}{b_0}\right]$.

4. Sketch the following real-valued functions over the interval $-4 \le x \le +4$ (you also may use a computer program, but it may get you into trouble and you will need to know how to sketch them!)

   (a) $f[x] = \cos\left[\pi x^2\right]$

   (b) $f\left[x + \frac{1}{2}\right]$

   (c) $g[x] = \cos\left[\pi\left(x^2 + \frac{1}{2}\right)\right]$

   (d) $f\left[\frac{2x}{3}\right]$

   (e) $h[x] = e^{-\pi(x-1)^3}$

   (f) $k[x] = e^{-\pi(x-1)^4}$

5. Sketch the following complex-valued functions over the interval $-4 \le x \le +4$ as (1) real part, (2) imaginary part, (3) magnitude, and (4) phase. Again, you may plot them by computer.

   (a) $f[x] = TRI\left[\frac{x}{2}\right] - i\,TRI\left[\frac{x}{2}\right]$

   (b) $g[x] = RECT\left[\frac{x-1}{2}\right] + i\,RECT\left[\frac{-x-1}{3}\right]$

   (c) $h[x] = e^{-(x-1)} STEP[x-1] + i\,e^{-(x+1)} STEP[x+1]$

   (d) $p[x] = (1+i)\ e^{+i\pi x^2}$

6. Find the area of $GAUS[x]$ by constructing the simultaneously separable *and* circularly symmetric 2-D Gaussian $e^{-\pi\left(x^2+y^2\right)}$ and evaluating area in polar coordinates by applying easily solved integrals.

7. Express the following product of two Gaussian functions as a Gaussian function, i.e., given $x_0 \geq 0$, evaluate A_0 and d_0 in the following expression:

$$GAUS\left[\frac{x+x_0}{b_0}\right]\cdot GAUS\left[\frac{x-x_0}{b_0}\right] = A_0\ GAUS\left[\frac{x}{d_0}\right]$$

8. With $f[x]$ an arbitrary function and b and x_0 real-valued constants, show that: (Gaskill 3-6)

$$f[x]\cdot COMB\left[\frac{x-x_0}{b}\right] = |b|\sum_{n=-\infty}^{+\infty} f[x_0+nb]\ \delta[x-x_0-nb]$$

9. Express the following product of two Gaussian functions as a Gaussian function, i.e., given $x_0 \geq 0$, evaluate A_0 and d_0 in the following expression:

$$GAUS\left[\frac{x+x_0}{b_0}\right]\cdot GAUS\left[\frac{x-x_0}{b_0}\right] = A_0\ GAUS\left[\frac{x}{d_0}\right]$$

10. OPTIONAL EXTRA CREDIT: In the notes, the area of $\exp\left[-\pi\left(\frac{|x|}{b}\right)^n\right]$ is evaluated by changing variables to put the area integral into the form of the gamma function. Evaluate the areas of the general forms

$$\begin{aligned} f_1[x] &= \cos\left[\pi\left(\frac{|x|}{b}\right)^n\right] \\ f_2[x] &= \sin\left[\pi\left(\frac{|x|}{b}\right)^n\right] \end{aligned}$$

and use them to evaluate the areas of the chirp functions.

$$\begin{aligned} f_1[x] &= \cos\left[\pi\left(\frac{|x|}{b}\right)^2\right] \\ f_2[x] &= \sin\left[\pi\left(\frac{|x|}{b}\right)^2\right] \end{aligned}$$