1051-716-20071 Homework Assignment #1 Due 9/12/2007 (W)

0. Review the concepts of complex numbers and their geometrical representation. Sources include text (Chapters 4,5), Gaskill (Chapter 2), Bracewell (Fourier Transform and its Applications , Chapter 2). Any of several math books on complex analysis may also be useful.

1. For $z_1 = 2 + 2i$, locate the following points in a single sketch of on the complex plane.

   (a) z_1
   (b) z_1^*
   (c) z_1^{-1}

2. For $z_1 = 2 + 2i$ as above and $z_2 = -\frac{1}{2} - \frac{1}{2}i$, , evaluate the operations and locate the resulting complex numbers on the complex plane. You may plot them on the same graph but be sure that their locations are obviously labeled.

   (a) $z_1 + z_2$
   (b) $z_1 - z_2$
   (c) $z_1 z_1^*$
   (d) $z_1 z_2$
   (e) $z_1 z_2^*$
   (f) $z_1^* z_2$
   (g) $z_1^* z_2^*$
   (h) $\frac{z_2}{z_1}$

3. Calculate all values of z that satisfy the following equations; express these values of z as both real/imaginary parts and as magnitude/phase.

   (a) $z^2 - i = 0$
   (b) $z^3 + i = 0$
   (c) $z^3 - 1 = i$
   (d) $z^2 + i = 9$

4. Find the complex numbers z that are complex conjugates of z^2.

5. Use complex analysis to demonstrate that:

   (a) $\cos[5\theta] = \cos^5[\theta] - 10\cos^3[\theta]\sin^2[\theta] + 5\cos[\theta]\sin^4[\theta]$
   (b) $\sin[5\theta] = \sin^5[\theta] - 10\cos^2[\theta]\sin^3[\theta] + 5\cos^4[\theta]\sin[\theta]$

6. For $f[x] = A\exp[+2\pi i\xi_0 x]$, where A and ξ_0 are real-valued and positive parameters, find and sketch the following functions of x:

   (a) $f[x]$
   (b) $f^*[x]$
   (c) $f[x] + f^*[x]$
   (d) $|f[x] + f^*[x]|^2$
   (e) $|f[x]|^2$

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7. For the 1-D function of x

$$f[x] = RECT\left[\frac{x+a}{b}\right] + RECT\left[\frac{x-a}{b}\right],$$

sketch $f[x]$ for the following parameters:

(a) $a = 2b$

(b) $a = b$

(c) $a = \frac{b}{2}$ (also write down a different and more concise expression for this function)

8. For the function $f[x] = RECT\left[x + \frac{1}{2}\right] + STEP[x] \cdot TRI[x]$:

(a) Determine the "support" and the "area" of $f[x]$

(b) Sketch $f\left[\frac{-x+1}{2}\right]$ and $f[-2x+4]$.

9. Evaluate the integral

$$\int_{-\infty}^{+\infty} (A_0 + A_1 \cos[2\pi\xi_0\beta]) \cdot RECT[x-\beta]\ d\beta$$

Since this is integrating over the "space" variable β, the answer is a function of x, A_0, A_1, and ξ_0.