1051-716-20111 Final Exam 17 November 2011

TURN OFF AND STOW cell phones, laptops, calculators
THREE HOURS: SELECT NINE 9 of the following 12 problems (equal weight); Start each problem on a new sheet; Staple and submit ONLY 9 problems in numerical order
SHOW AND EXPLAIN YOUR WORK; STATE ANY THEOREMS YOU USE
Usual Hints: Make sketches before writing equations; if a problem seems overly difficult, you probably should think a different way; if the problem seems simple, use caution because you may have missed something.
Possibly Useful Information:

$$1 = \int_{-\infty}^{+\infty} \delta[x]\;dx = \int_{-\infty}^{+\infty} RECT[x]\;dx = \int_{-\infty}^{+\infty} TRI[x]\;dx = \int_{-\infty}^{+\infty} \exp[-x]\cdot STEP[x]\;dx$$

$$= \int_{-\infty}^{+\infty} SINC[x]\;dx = \int_{-\infty}^{+\infty} SINC^2[x]\;dx = \int_{-\infty}^{+\infty} \exp\left[-\pi x^2\right]\;dx = \int_{-\infty}^{+\infty} \frac{J_0[x]}{2}\;dx = 1$$

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1. (Essay) In class, I stated that the recipes for the inverse and matched filters are related, even though the applications seem to be quite different. Write a few paragraphs with appropriate equations that compare and contrast both the applications and the recipes for the two filters. Show how the form of the "approximate" filter for one application may be applied to the other. Feel free to use sketches to illustrate your points.

2. A general 1-D complex valued function $f[x] = \mathrm{Re}\{f[x]\} + i\cdot\mathrm{Im}\{f[x]\}$ may be decomposed into four components based on symmetry (even or odd) and complex character (real or imaginary).

   (a) Write down the equations for deriving the four component parts of $f[x]$.

   (b) Determine the character (real, imaginary, or complex and even, odd, or neither) the Fourier transform of each of the four component parts.

3. Consider the 2-D convolution:

$$p[x,y] = \left(J_0\left[2\pi\sqrt{\frac{x^2+y^2}{4}}\right]\cdot\exp\left[+i\pi\frac{x}{2}\right]\right) * \left(J_0\left[2\pi\sqrt{\frac{x^2+y^2}{4}}\right]\cdot\exp\left[-i\pi\frac{x}{2}\right]\right)$$

   Evaluate the result of this convolution up to a constant; in other words, you need derive only the *form* of the function, not its exact amplitude. *HINT: I think that sketch(es) are essential to solve this problem.*

4. Determine the area of $f[x] = (SINC[x])^4$

5. Evaluate and graph the real part, imaginary part, magnitude, and phase of $\mathcal{F}_1\left\{-i\cdot\frac{1}{1+x^2}\right\}$

6. Evaluate and sketch the autocorrelation of $f[x] = (2+2i)\cdot\exp\left[+i\pi\left(\frac{x}{2}\right)^2\right]$

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7. Consider the input signal $f[x] = COMB\left[\frac{x}{2}\right]$ which is filtered by convolving with $h[x] = RECT[x]$ to produce $g[x]$.

   (a) Evaluate and sketch $g[x]$.

   (b) Specify and sketch the transfer function $W[\xi]$ of the inverse (or, if more appropriate, the pseudoinverse) filter for $h[x]$.

   (c) Evaluate and sketch the estimate of $f[x]$ that would be recovered by applying $w[x] = \mathcal{F}_1^{-1}\{W[\xi]\}$ to $g[x]$.

   (d) Explain any artifacts present in the estimate of $f[x]$.

8. Derive $f[x,y]$ if $F[\xi.\eta] = \left(\sqrt{\left[(\xi-1)^2 + (\eta+1)^2\right]}\right)^{-1}$; show the steps in the derivation, do not just write down a result.

9. Evaluate the 2-D convolution $RECT\left[\frac{x}{2}, y\right] * (\delta[x] \cdot 1[y] + 1[x] \cdot \delta[y])$ and sketch the profiles along the x and y axes.

10. For $s[x] = COMB[x]$, $h[x] = SINC[x]$, and $f_n[x] = SINC^2\left[\frac{x}{n}\right]$ $(n = 1, 2, 3)$, find the forms of and sketch the three functions $g_n[x] = (f_n[x] \cdot s[x]) * h[x]$

11. Consider the function $f_1[x] = \frac{1}{4} \cdot COMB\left[\frac{x-2}{4}\right]$

    (a) Evaluate and sketch its Fourier transform $F_1[\xi]$.

    (b) Evaluate and sketch the Fourier transform of $f_2[x] = \frac{1}{4} \cdot COMB\left[\frac{x+2}{4}\right]$

    (c) Compare the sketches of the transforms $F_1[\xi]$ and $F_2[\xi]$; what does the comparison tell you about $f_1[x]$ and $f_2[x]$?

12. Two imaging systems have respective transfer functions $H_1[\xi,\eta] = RECT[\xi,\eta]$ and $H_2[\xi,\eta] = CYL\left(\sqrt{\xi^2+\eta^2}\right)$.

    (a) Sketch "top views" (NOT "profiles") of the transfer functions $H_1[\xi,\eta]$ and $H_2[\xi,\eta]$.

    (b) Evaluate the impulse responses $h_1[x,y]$ and $h_2[x,y]$.

    (c) Make an approximate sketch in the $[x,y]$ domain of the locations of the first zeros of the two impulse responses, i.e., draw lines in the $[x,y]$ plane that show the locations closest to the origin where $h_1[x,y] = 0$ and $h_2[x.y] = 0$. Be sure that the lines belonging to each are labeled unambiguously.

    (d) (OPTIONAL BONUS) Which of these two systems will have the better resolution? Explain.