

**TURN OFF** and stow cell phones, laptops, and other communications devices. Calculators not allowed (and not needed)

**TWO HOURS: SELECT SEVEN** 7 of the following problems (equal weight);

Start each problem on a new sheet; Staple and submit 7 problems in numerical order

**SHOW/EXPLAIN YOUR WORK: YOU MAY USE THEOREMS WITHOUT PROOF, BUT STATE WHAT YOU USE**

Usual Hints: Make sketches before writing equations; if a problem seems overly difficult, you probably should think a different way; if the problem seems simple, use caution because you may have missed something.

1. For  $f[x] = RECT\left[\frac{x+1}{2}\right] - TRI\left[\frac{x-2}{2}\right]$

- (a) sketch  $f[x]$
- (b) evaluate the area of  $f[x]$
- (c) evaluate and sketch the even and odd parts of  $f[x]$

2. Find all values of  $z$  that satisfy the condition  $(z^*)^{\frac{1}{2}} = z^2$

3. Use the definition of the Dirac delta function to evaluate AND sketch  $\delta[-SINC[x]]$

4. Evaluate the area of:

$$f[x] = \sum_{n=-5}^{+5} \cos\left[\pi \frac{(x+n)^2}{4}\right]$$

5. Evaluate and sketch the output of the convolution:

$$f[x] = \sin[2\pi\xi_0 x] * RECT\left[\frac{x}{b_0}\right] \text{ where } b_0 = 2(\xi_0^{-1})$$

6. Evaluate and sketch the output of the convolution:

$$r[x] = STEP[x] * \left(\exp\left[-\frac{x}{2}\right] \cdot STEP[x]\right)$$

7. Evaluate and sketch the output of the convolution::

$$s[x] = SINC[2x] * SINC^2\left[\frac{x}{2}\right]$$

8. Given that  $\mathcal{F}_1\{f[x]\} = F[\xi]$  and  $\mathcal{F}_1\{m[x]\} = M[\xi]$

- (a) Derive an expression for  $\mathcal{F}_1\{f[x] \cdot m[x]\}$
- (b) Evaluate and sketch the spectrum if  $f[x] = SINC[x]$  and  $m[x] = COMB[4x]$

9. For  $F[\xi] = (1+i) \cdot \delta[\xi+1] + (1-i) \cdot \delta[\xi-1]$

- (a) Find an expression for  $f[x]$
- (b) Sketch  $f[x]$  as real part, imaginary part, magnitude, and phase.
- (c) **OPTIONAL BONUS**, find an expression for  $g[x]$  if  $G[\xi] \equiv F[\xi] * \delta[\xi-1]$ ; sketch  $g[x]$ ,  $|g[x]|$ , and  $\Phi\{g[x]\}$ .