1051-716-20091 Midterm Exam 14 October 2009

SELECT SIX (6) of the following seven problems (equal weight)
PENCIL/PEN AND PAPER ONLY (CLOSED BOOK, NO NOTES)
NO CALCULATORS SHOW YOUR WORK!
SUBMIT IN NUMERICAL ORDER ONLY THOSE 6 YOU HAVE SELECTED.

1. The "ramp" function may be defined:

$$f[x] = RAMP[x] \equiv x \cdot STEP[x]$$

   (a) Sketch $f[x] = RAMP[x]$

   (b) Find expressions for the even and odd parts of $f[x]$ AND sketch them.

   (c) Evaluate $f[x] * f[x]$ AND sketch it.

   (d) Evaluate $f[x] \bigstar f[x]$.

2. Evaluate the integral and sketch the output(s):

$$\int_{-\infty}^{+\infty} \cos[2\pi\xi_0 x + \phi_0] \cdot (\cos[2\pi\xi x] + \sin[2\pi\xi x])\ dx$$

   where ξ_0 and ϕ_0 are real-valued numerical constants.

3. For TWO (2) of the following three expressions (your choice), find and sketch the Argand diagram of the sets of complex numbers z that satisfy these sets of conditions independently:

   (a) $z^4 = 1 + i$

   (b) $z^3 = -8$

   (c) $z^{\frac{1}{2}} = -2 + 3i$

4. Evaluate THREE (3) of the following expressions (your choice); you may use any expressions derived in class without proof, but state what you use.

   (a) $f[x] = \cos[2\pi\xi_0 x + \phi_0] * RECT\left[\frac{x}{b_0}\right]$ where ξ_0, b_0, and ϕ_0 are positive real numbers.

   (b) $g[x] = \cos[2\pi\xi_0 x + \phi_0] * TRI\left[\frac{x}{b_0}\right]$ where ξ_0, b_0, and ϕ_0 are positive real numbers.

   (c) $r[x] = (e^{-x} \cdot STEP[x]) \bigstar (e^{-x} \cdot STEP[x])$

   (d) $s[x] = (RECT[x+1] + RECT[x-1]) * STEP[-x]$

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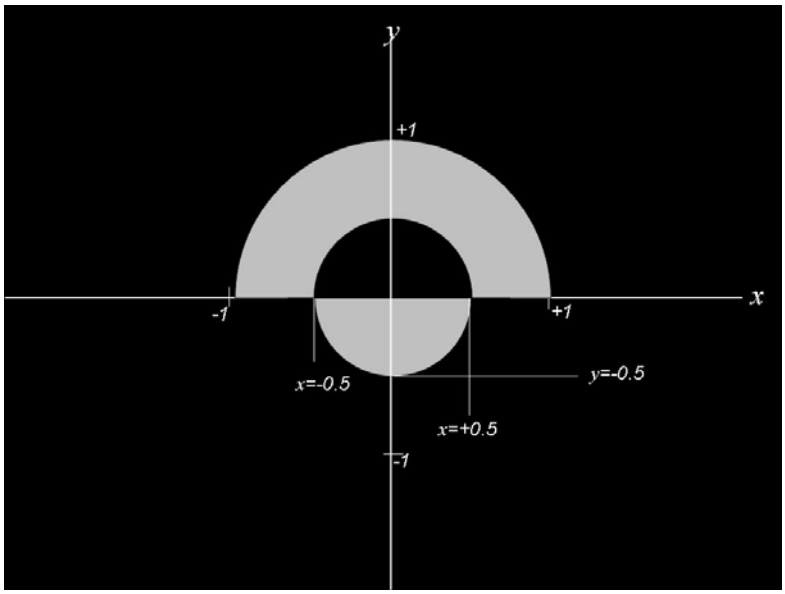
5. For the real-valued 2-D function shown below, gray corresponds to amplitude $\mathrm{Re}\{f[x,y]\} = 0.5$ and black to $\mathrm{Re}\{f[x,y]\} = 0$.



   (a) Determine the limits of the region of support of the autocorrelation.

   (b) Evaluate the amplitude of the autocorrelation at $[x,y] = [0,0]$.

   (c) Determine the answers to (a) and (b) if $f[x,y]$ were purely imaginary, i.e., if $\mathrm{Re}\{f[x,y]\} = 0\,[x,y]$ and the imaginary part is as shown.

   (d) OPTIONAL, EXTRA CREDIT: write down the functional form of the real-valued function $f[x,y]$ as the SUM OF TWO (and only two) terms, where each term may be the product of two (or more) special functions.

6. For ALL of the following, sketch the functions and determine their areas or volumes as appropriate:

   (a) $\delta[x^3 - x]$

   (b) $\delta[\underline{\mathbf{r}} - \underline{\mathbf{r}}_0]$

   (c) $\delta[r - r_0]$

7. Sketch the following functions and find expressions for their even and odd parts.

   (OPTIONAL EXTRA CREDIT: sketch the even and odd parts in either or both cases).

   (a) $f[x,y] = \delta[x-1, y-1] * \delta\left(r - \sqrt{2}\right)$

   (b) $g[x] = \cos[\pi(x^2 - 2x + 1)]$ (Hint: simplify before sketching)