

SIMG-716 Fourier Methods for Imaging

This course presents mathematical descriptions for functions and systems and demonstrates their application to solving imaging problems.

Course Outline: (dates are estimates and subject to change, some material is tentative)

Week 1

- 1) Introduction and motivation
 - a) Three imaging “entities:” input object $f[x,y]$, output image $g[x',y']$, system function $h[x,y;x',y']$
 - b) Three imaging problems (direct, inverse, system analysis/synthesis)
 - c) Examples of imaging systems and models
 - d) Necessity to constrain action of system for tractable mathematical description
- 2) Complex numbers and their geometric interpretation
 - a) complex numbers as real-valued vectors
 - b) representations: real+imaginary, magnitude+phase, phasor/Argand diagram
 - c) complex arithmetic
 - d) Euler relation, deMoivre's theorem, roots of complex numbers

Week 2

- 3) Functions
 - a) classifications
 - i) domain and range (real/complex, continuous/discrete)
 - ii) form (linear/nonlinear, periodic, harmonic)
 - iii) symmetry (even/odd)
 - b) Representations obtained by projecting onto different sets of basis functions
 - i) inner product, relation to scalar product
 - ii) orthogonal/orthonormal sets of functions
 - iii) power series representations, Taylor series
 - c) 1-D real-valued functions: unit-amplitude function $1[x]$, null function $0[x]$, rectangle, triangle, signum, step, signum, cosine, chirp, Gaussian functions
 - d) 1-D Dirac delta function (impulse) and related functions
 - e) 1-D complex-valued functions
 - f) 1-D stochastic functions (uniform, Gaussian, and Poisson distributions)
- 4) 2-D functions from 1-D functions
 - a) separable in Cartesian coordinates
 - b) separable in polar coordinates \Rightarrow circular symmetry
 - c) 2-D Dirac delta function
 - i) representations in Cartesian and polar coordinates
 - ii) variants of 2-D Dirac delta function (line delta and *CROSS*)
 - d) representations of rotated 2-D functions

Week 3

- 5) Mathematical representation of systems, operators
 - a) Linearity property
 - b) Shift (space-, time-) variance and shift (space-, time-) invariant property
 - c) linear and shift-invariant (LSI) systems, convolution/filtering
 - d) Representations of systems
 - i) linear discrete \Rightarrow analogy with matrix -vector multiplication

- ii) linear continuous \Rightarrow superposition integral
- iii) LSI discrete systems \Rightarrow circulant matrix, diagonalizing transformation
- iv) LSI continuous systems \Rightarrow convolution integral
- v) impulse response/point-spread function of LSI continuous system
- e) Crosscorrelation and autocorrelation as variants of convolution

Week 4

- 6) Alternative representations of functions
 - a) Projection of functions onto alternative sets of basis functions
 - i) representation in space and spatial frequency domain
 - b) Intuitive derivation of sine and cosine transforms
 - c) Hartley transform as weighted sum of cosine and sine transforms
 - d) Fourier transform as weighted sum of cosine and sine transforms
 - e) Fourier series for discrete functions

Week 5

- 7) Fourier transforms of 1-D functions
 - a) Evaluation of Fourier transforms by direct integration
 - b) Evaluation of Fourier transforms of special functions
 - c) Theorems of the Fourier transform
 - d) Chirp Fourier transform

MIDTERM EXAM, 10/12 (W)**Week 6**

- 8) Fourier transforms of 2-D functions
 - a) Separable functions in Cartesian coordinates
 - b) Transforms of circularly symmetric functions (Hankel transform)
 - c) Radon transform

Week 7

- 9) Discrete functions
 - a) Ideal sampling at uniform intervals, sampled special functions
 - b) Interpretation of sampling in frequency domain, Whittaker-Shannon sampling theorem
 - c) Realistic sampling at uniform intervals, reduced modulation
 - d) Reconstruction of functions from samples, interpolation
- 10) Processing of discrete functions
 - a) Discrete Fourier transform (DFT)
 - i) 1-D DFT as matrix applied to 1-D discrete vector
 - ii) inverse DFT, normalization conventions
 - iii) Fourier series from the Fourier transform
 - b) Computation of the DFT
 - c) computation of spectrum at arbitrary frequency
 - d) Cooley-Tukey implementation of DFT \Rightarrow "fast Fourier transform (FFT)
 - e) 2-D FFT

Week 8

- f) Practical Considerations of DFT
 - i) leakage, window functions
 - ii) Resolution in space and frequency domains, zero padding
 - iii) Data formats, centered and noncentered data, checkerboarding
 - iv) Phase in the FFT
- g) convolution of discrete functions
- h) normalization conventions
- i) linear and circular convolution

Week 9

11) Linear Filtering

- a) impulse response and transfer function, psf and OTF
- b) Amplitude filters (lowpass, highpass, bandpass, bandstop)
- c) Allpass (phase-only) filters
- d) Deconvolution and detection filters
 - i) Inverse filter
 - ii) Wiener and Wiener-Helstrom filter
 - iii) Matched filter

Week 10

12) Applications of linear systems in imaging

- a) Fresnel diffraction as convolution with chirp function
- b) Implementation of optical chirp Fourier transform
- c) Radon transform as basis for medical computed tomography (CT) and magnetic resonance imaging (MRI)

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