

SIMG-716 Fourier Methods for Imaging

This course presents mathematical descriptions for functions and systems and demonstrates their application to solving imaging problems.

Course Outline: (dates are estimates and subject to change, some material is tentative)

Week 1

- 1) Introduction and motivation
 - a) The three imaging problems (direct, inverse, system analysis/synthesis)
 - b) Examples of imaging systems and models
 - c) Necessity to constrain action of system to enable tractable description

- 2) Complex numbers and their geometric interpretation
 - a) complex numbers as real-valued vectors
 - b) representations: real+imaginary, magnitude+phase, phasor/Argand diagram
 - c) complex arithmetic
 - d) Euler relation, deMoivre's theorem

Week 2

- 3) Functions
 - a) classifications
 - i) domain and range (real/complex, continuous/discrete)
 - ii) form (linear/nonlinear, periodic, harmonic)
 - iii) symmetry (even/odd)
 - b) Representations obtained by projecting onto different sets of basis functions
 - i) inner product, relation to scalar product
 - ii) orthogonal/orthonormal sets of functions
 - iii) power series representations, Taylor series
- 4) 1-D real-valued functions
 - a) $I[x]$, $O[x]$, *RECT*, *TRI*, *SGN*, *SINC*, *STEP*, *COS*, *CHIRP*, *GAUS*, *LOR*
- 5) 1-D Dirac delta function (impulse) and related functions
- 6) 1-D complex-valued functions
- 7) 1-D stochastic functions (uniform, Gaussian, Poisson)
- 8) 2-D functions from 1-D functions
 - a) separable in Cartesian coordinates
 - b) separable in polar coordinates = circularly symmetric
 - c) 2-D Dirac delta function
 - i) variants of the 2-D Dirac delta function (line delta, *CROSS*, etc.)
 - d) rotated 2-D functions

Week 3

- 9) Mathematical representation of systems, operators
- a) Linearity
 - b) Shift (space-, time-) variance and invariance
 - c) linear and shift-invariant (LSI) systems, convolution/filtering
 - d) Representations of systems
 - i) linear discrete -- matrix-vector multiplication
 - ii) linear continuous -- superposition integral
 - iii) LSI discrete -- circulant matrix, diagonalizing transformation
 - iv) LSI continuous -- convolution integral
 - v) impulse response/point-spread function
 - e) Crosscorrelation and autocorrelation

Week 4

- 10) Alternative representations of functions
- a) Projections onto other bases
 - i) representation in space and spatial frequency domain
 - b) Intuitive derivation of sine and cosine transforms
 - c) Hartley transform
 - d) Fourier transform
 - e) Fourier series

Week 5

- 11) Fourier transforms of 1-D functions
- a) Direct integration, Fourier transforms of special functions
 - b) Theorems of the Fourier transform
 - c) Chirp Fourier transform

MIDTERM EXAM, 10/15 (W)**Week 6**

- 12) Fourier transforms of 2-D functions
- a) Separable functions in Cartesian space
 - b) Circularly symmetric functions (Hankel transform)
 - c) Radon transform

Week 7

13) Discrete functions

- a) Ideal sampling at uniform intervals, sampled special functions
- b) Interpretation of sampling in frequency domain, Whittaker-Shannon sampling theorem
- c) Realistic sampling at uniform intervals, reduced modulation
- d) Reconstruction of functions from samples, interpolation

14) Processing of discrete functions

- a) Discrete Fourier transform
 - i) 1-D DFT as matrix
 - ii) inverse DFT, normalization conventions
 - iii) Fourier series from the Fourier transform
- b) Computation of the DFT
 - i) computation at arbitrary frequency
 - ii) Cooley-Tukey FFT
 - iii) 2-D FFT

Week 8

- c) Practical Considerations
 - i) leakage, window functions
 - ii) Resolution in space and frequency domains, zero padding
 - iii) Data formats, centered and noncentered data, checkerboarding
 - iv) Phase in the FFT
- d) convolutions of discrete functions
 - i) normalization conventions
 - ii) linear and circular convolution

Week 9

15) Linear Filtering

- a) impulse response and transfer function, psf and OTF
- b) Amplitude filters (lowpass, highpass, bandpass, bandstop)
- c) Allpass (phase-only) filters
- d) Deconvolution and detection filters
 - i) Inverse filter
 - ii) Wiener and Wiener-Helstrom filter
 - iii) Matched filter

Week 10

16) Applications of linear systems in imaging