6 November 2009 ①

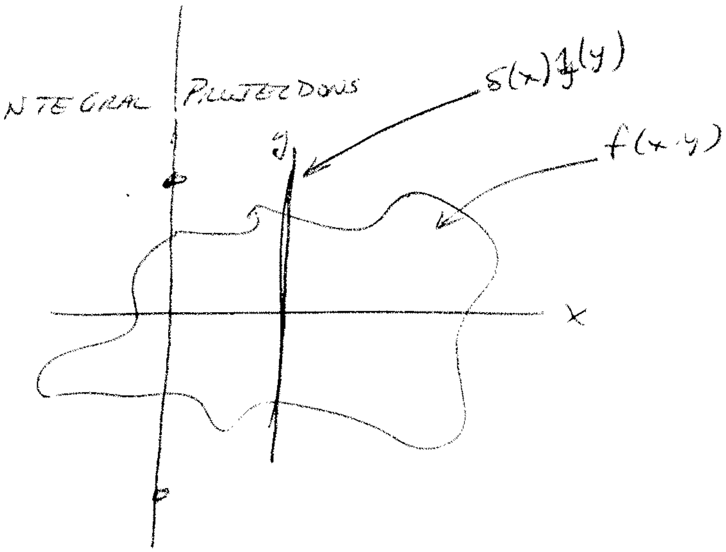
Problem / Question Session

Radon Transform — line-integral projections

$f[x,y] ** \delta(x')\,1(y')$

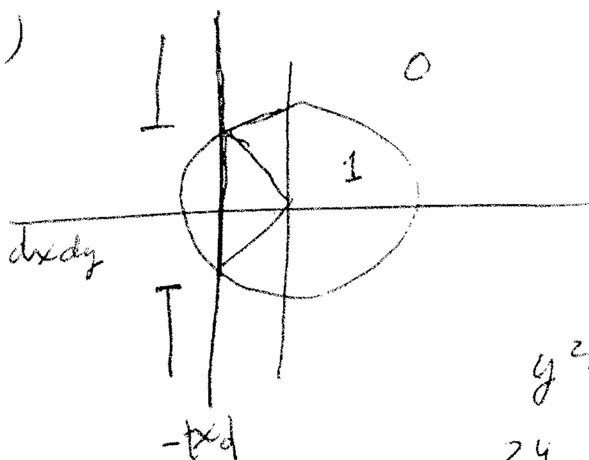
Output is constant in y

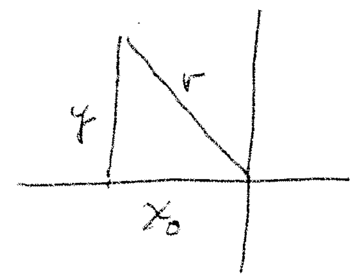
function of x only



$\underline{CYL(r)} ** \delta(x)\,1(y)$

$\iint CYL(r)\cdot\delta(x+|x_0|)1(y)\,dx\,dy$





$y^2 = r^2 - x_0^2$

$2y = 2\sqrt{r^2 - x_0^2}$

$y = \sqrt{4r^2 - x_0^2}$

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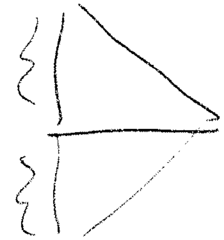
$$\mathrm{CYL}\left(\frac{r}{1}\right) * \delta(x)\, 1(y) = 1(y) \cdot \begin{cases} \sqrt{1-4x^2} & \text{IF } |x| < \frac{1}{2} \\ 0 & \text{IF } |x| > \frac{1}{2} \end{cases}$$

$$= \sqrt{1-4x^2} \cdot \mathrm{RECT}[x] \cdot 1(y)$$



$$\mathrm{CYL}\left(\frac{r}{2}\right) * \delta(x)\, 1(y) = 4\sqrt{1-x^2} \cdot 1(y) \cdot \mathrm{RECT}\left(\frac{x}{2}\right)$$

$$2y = 2\sqrt{r^2-x^2}$$
$$= 2\sqrt{1-x^2}$$

$r=1$ $\quad 2\sqrt{1-x^2}$

$r=1/2$ $\quad 2\sqrt{\frac{1}{4}-x^2} = \sqrt{1-4x^2}$

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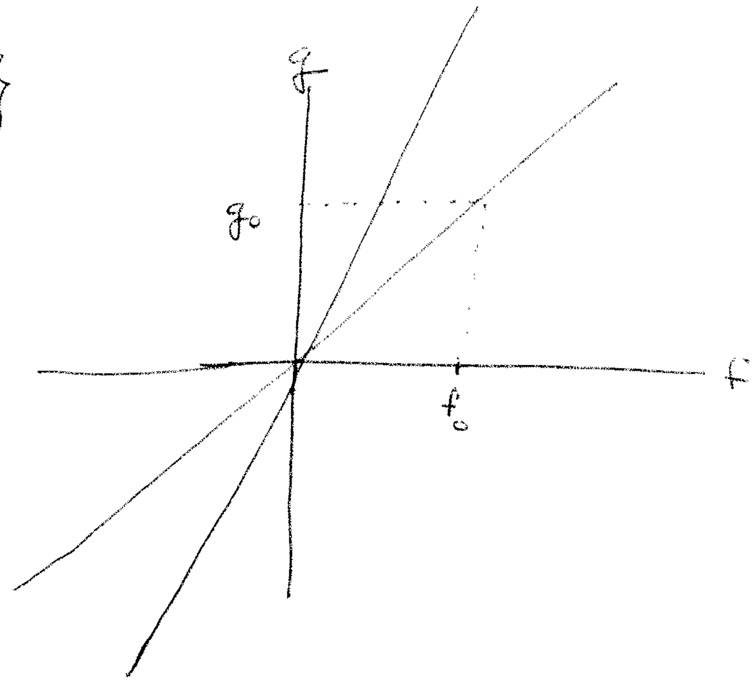
$g(x) = \mathcal{N}\{f(x)\}$



— $g_0 = f_0$

— $g_0 = 2f_0$

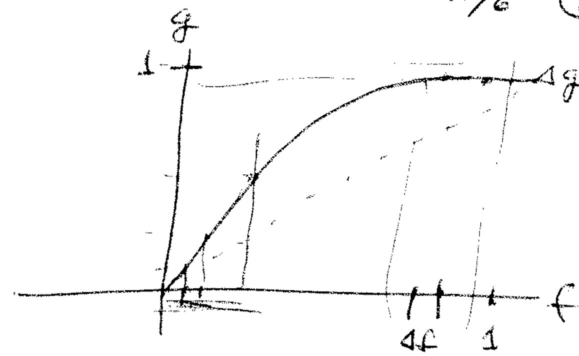
$g_0 = k \cdot f_0$

$\mathcal{F}\{f(x)\} = F(\xi)$

$\mathcal{F}\{g(x)\} = \mathcal{F}\{k \cdot f(x)\} = k \cdot F(\xi)$ LINEAR, NO NEW FREQUENCIES

$$g = \sin\left(f \cdot \frac{\pi}{2}\right) \quad \text{if} \quad 0 \le f \le 1$$

$$0 \quad \text{otherwise}$$

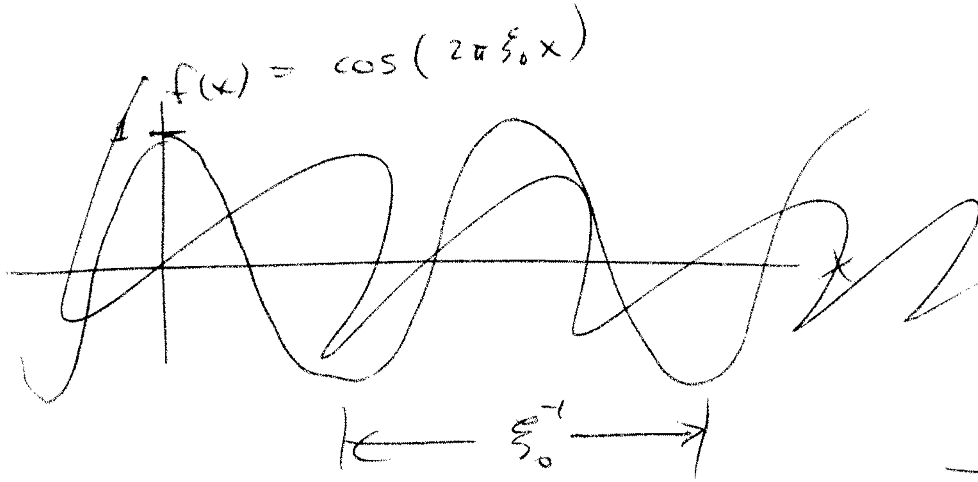


~~EXPANDING~~ CONTRAST OF SHADOWS

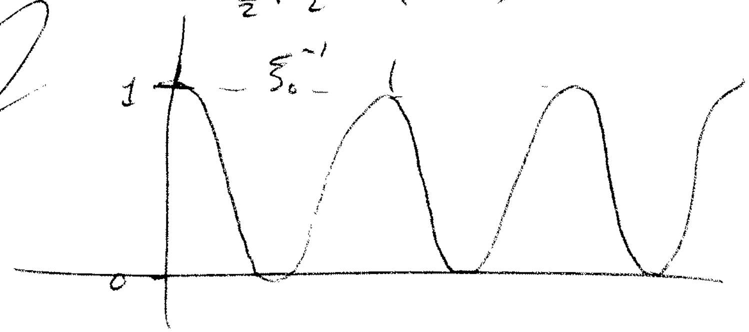
~~CNT~~
COMPRESSING CONTRAST OF HIGHLIGHT

$$\mathcal{F}\{f(x)\} = F(\xi)$$

$$f(x) = \cos(2\pi \xi_0 x)$$



$$\frac{1}{2} + \frac{1}{2}\cos(2\pi \xi_0 x)$$



$$g = \sin\left(f \cdot \frac{\pi}{2}\right)$$

$$\sin(\theta) = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \cdots$$

$$g(x) = \underline{\frac{\pi}{2} \cdot f(x)} - \frac{\left(\frac{\pi}{2}\right)^3}{6}\left(f(x)\right)^3 + \left(\frac{\pi}{2}\right)^5 \cdot \frac{1}{120} \cdot \left(f(x)\right)^5 - \cdots$$

$$G(\xi) = \frac{\pi}{2} \cdot F(\xi) - \frac{\pi^3}{48} \mathcal{F}\left\{\underline{\left(f(x)\right)^3}\right\} + \frac{\pi^5}{32 \cdot 120} \mathcal{F}\left\{\left(f(x)\right)^5\right\} - \cdots$$

$$\uparrow$$

$$\frac{\pi}{2}\left(\frac{1}{2}\delta(\xi) + \frac{1}{4}\delta(\xi+\xi_0) + \frac{1}{4}\delta(\xi-\xi_0)\right)$$

$$\left(f(x)\right)^3 = \left(\frac{1}{2} + \frac{1}{2}\cos(2\pi\xi_0 x)\right)^3 = \frac{1}{8}\left(1 + \cos(2\pi\xi_0 x)\right)^3$$

$$= \frac{1}{8}\left(1 + 3\cos(2\pi\xi_0 x) + 3\cos^2(2\pi\xi_0 x) + \cos^3(2\pi\xi_0 x)\right)$$

$$\cos^2(2\pi\xi_0 x) = \frac{1}{2}\left(1 + \cos(2\pi \cdot 2\xi_0 x)\right)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

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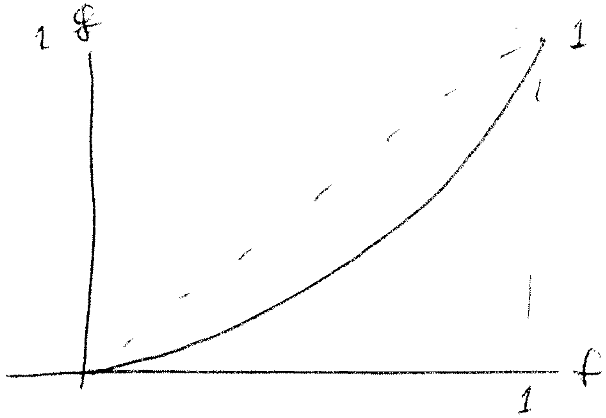

$$g = f^2 \qquad 0 \le f \le 1$$

$$f(x) = \frac{1}{2} + \frac{1}{2}\cos(2\pi\xi_0 x) \longrightarrow \left(\frac{1}{2} + \frac{1}{2}\cos(\;)\right)^2$$

$$g(x) = \left(f(x)\right)^2 = \underline{\frac{1}{4} + \frac{1}{4}\Big(}\frac{1}{2} + \frac{1}{2}\cos(2\pi\cdot 2\xi_0 x)\Big) + \underline{\cancel{\tfrac{1}{2}}\cos(2\pi\xi_0 x)}$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8}\cos(2\pi\cdot 2\xi_0 x) + \frac{1}{2}\cos(2\pi\xi_0 x)$$

$$= \frac{3}{8} + \frac{1}{2}\cos(2\pi\xi_0 x) + \frac{1}{8}\cos(2\pi\cdot 2\xi_0 x)$$

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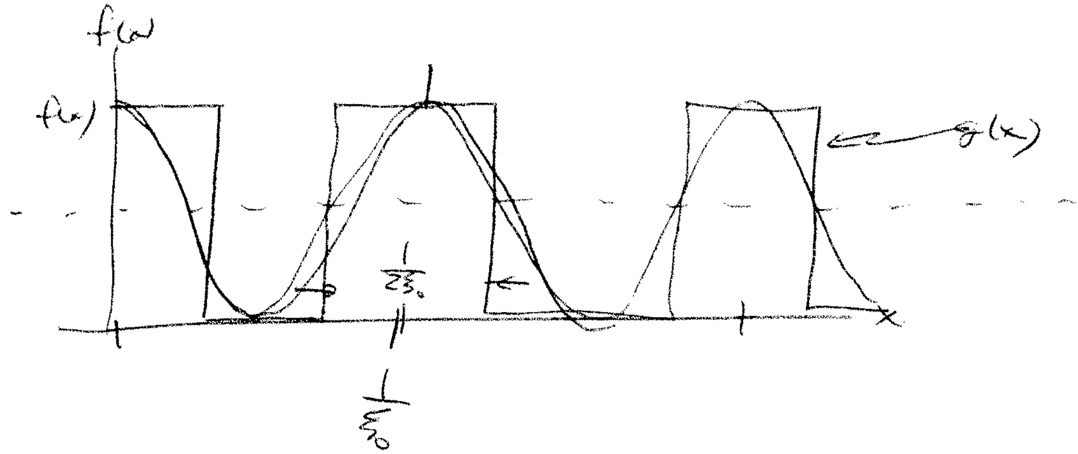
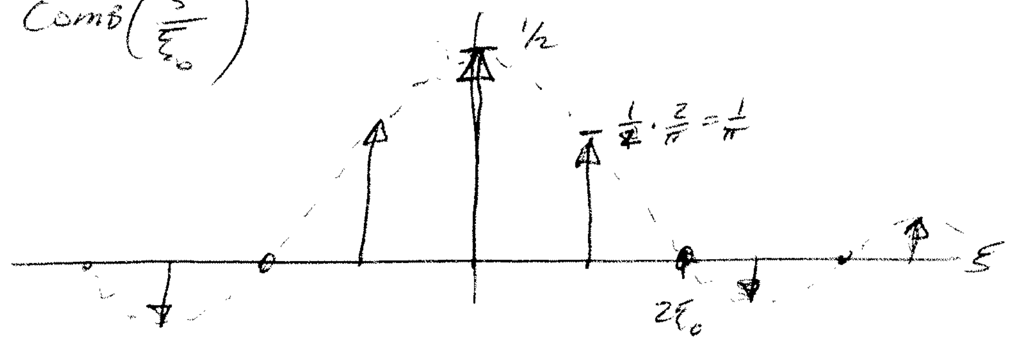


$$f(x) = \frac{1}{2} + \frac{1}{2}\cos(2\pi\xi_0 x) \implies F(\xi) = \frac{1}{2}\delta(\xi) + \frac{1}{4}\delta(\xi+\xi_0) + \frac{1}{4}\delta(\xi-\xi_0)$$

$$g(x) = \mathrm{STEP}(\cancel{f} \; f(x) - \tfrac{1}{2}) = \mathrm{RECT}\left[\frac{x}{1/2\xi_0}\right] * \left|\frac{1}{1/\xi_0}\right| \mathrm{COMB}\left(\frac{x}{1/\xi_0}\right)$$

$$= \mathrm{RECT}[2\xi_0 x] * |\xi_0|\, \mathrm{COMB}(\xi_0 x)$$

$$G(\xi) = \frac{1}{2\xi_0}\mathrm{SINC}\left(\frac{\xi}{2\xi_0}\right) \cdot \mathrm{COMB}\left(\frac{\xi}{\xi_0}\right)$$



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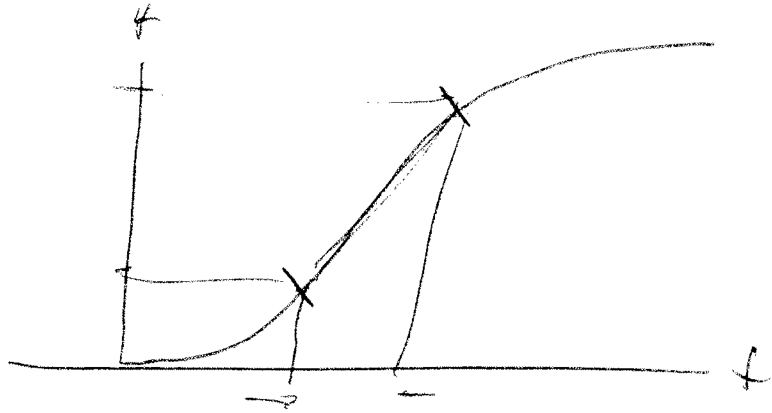
$$1 \,\mathrm{comb}(\xi_0 x) = \sum_{n=-\infty}^{+\infty} \delta(\xi_0 x - n)$$

$$= \sum_n \delta\left(\xi_0\left(x - \frac{n}{\xi_0}\right)\right) = \frac{1}{|\xi_0|} \sum_n \delta\left(x - \frac{n}{\xi_0}\right)$$

$$|\xi_0| \,\mathrm{comb}(\xi_0 x) = 1 \sum_n \delta\left(x - \frac{n}{\xi_0}\right)$$



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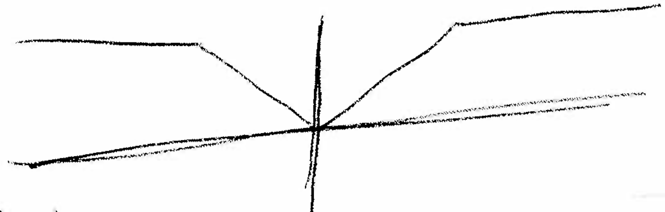
$$f(x) * \underbrace{h_1(x) * h_2(x)} = f(x) * h(x)$$

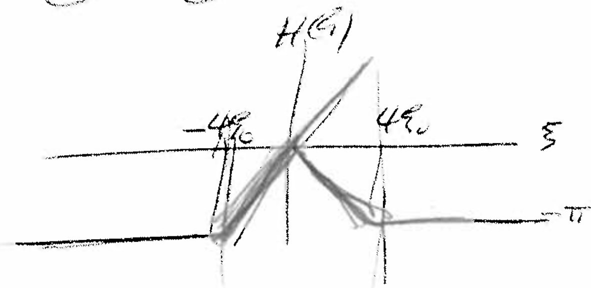
$$H(\xi) = H_1(\xi) \cdot H_2(\xi)$$

$$h_1(x) * h_2(x) * \cdots * h_N(x) \cong \frac{1}{d_0}\, \mathrm{gaus}\left(\frac{x}{b_0}\right)$$

CENTRAL LIMIT THEOREM § 13.3

PHASE FILTERING





$$e^{-i\pi\left(1-\mathrm{TRI}\left(\frac{\xi}{4\xi_0}\right)\right)} = H(\xi)$$

$$\left(1 - \mathrm{RECT}\left(\frac{\xi}{8\xi_0}\right)\right)(-1)$$

$$-1 + \mathrm{RECT}\left(\frac{\xi}{8\xi_0}\right) \cdot \left(\xi \cdot -\mathrm{SGN}(\xi)\right)$$

$\xi \cdot -\mathrm{SGN}(\xi)$

$$A_0 + A_1 \cos(2\pi\xi_0 x + \varphi_0)$$

$$= A_0 + \underbrace{(A_1 \cos\varphi_0)}_{\text{Amp}} \underbrace{\cos(2\pi\xi_0 x)}_{\text{even}} + \underbrace{(-A_0 \sin\varphi_0)}_{\text{Amp}} \underbrace{\sin(2\pi\xi_0 x)}_{\text{ODD}}$$

$$= A_0\,\delta(\xi) + \frac{A_1\cos\varphi_0}{2}\Big(\delta(\xi+\xi_0) + \delta(\xi-\xi_0)\Big) + -\frac{A_0\sin\varphi_0}{2}\cdot i\Big(\delta(\xi+\xi_0) - \delta(\xi-\xi_0)\Big)$$