2 November 2009

(1)

Office Hour / Problem Session This Friday 11/6 4 pm

Not following Friday 11/13

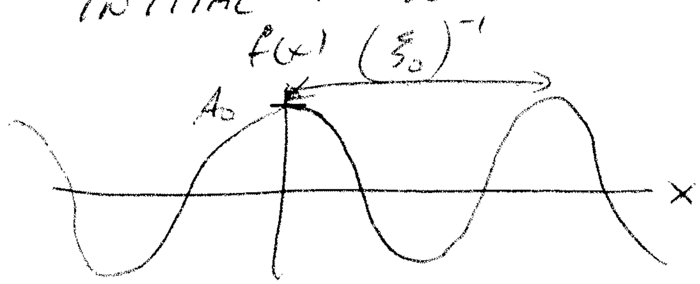
(May reschedule – possibly weekend or early in exam week)

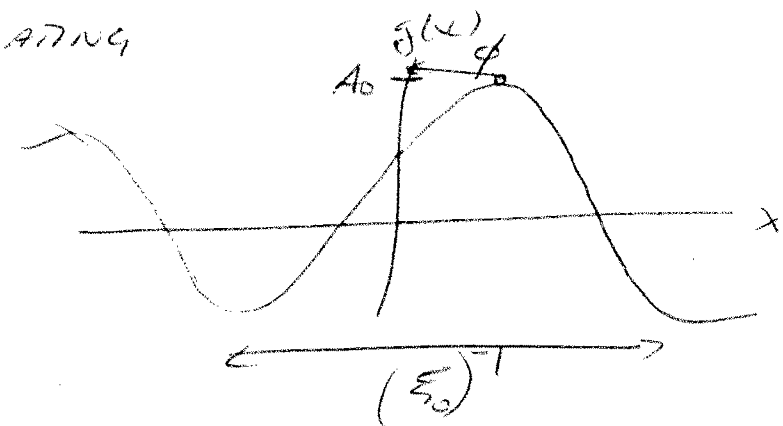
Final Exam on Th 11/19 1–4 pm (Two-hour test target)

Filtering ⇒ Phase Filtering = Allpass Filtering

$$H[\xi] = 1(\xi)\, e^{+i\phi_H(\xi)} = \cos(\phi(\xi)) + i \sin(\phi(\xi))$$

Does not amplify (or attenuate) components, & changes initial phase ⇒ translating





Filters with nonlinear phase significantly degrade output image

$$H(\xi) = 1(\xi)\, e^{+i\phi(\xi)}$$

$$|H(\xi)| = 1(\xi) = |H(\xi)|^2 \overset{\mathcal{F}^{-1}}{\Longrightarrow} \boxed{h(x) \star h(x) = \delta(x)}$$

$$g(x) = f(x) * h(x) \qquad G(\xi) = F(\xi) \cdot H(\xi)$$

$$|G(\xi)| = |F(\xi)| \cdot 1$$

$$g(x) \star g(x) = f(x) \star f(x) \xleftarrow{\mathcal{F}^{-1}} |G(\xi)|^2 = |F(\xi)|^2$$

AUTOCORRELATION IS PRESERVED BY APF

$$e^{+i\phi(\xi)} = e^{+i\pi w(\xi)} \qquad w(\xi) = \frac{\phi(\xi)}{\pi}$$

$$H(\xi) = \sum_{n=0}^{\infty} \frac{(+i\pi w(\xi))^n}{n!}$$

IF $w(\xi)$ IS SMOOTHLY VARYING (NO DISCONTINUITIES/DELTAS)

$$w(\xi) = a_0 + (a_1 \xi) + (a_2 \xi^2) + (a_3 \xi^3) + \cdots$$

$$\frac{1}{n!} \left. \frac{d^n w}{d\xi^n} \right|_{\xi=0}$$

$$\equiv \alpha_0 + \alpha_1 \xi + \underline{\underline{(\alpha_2 \xi)^2}} + (\alpha_3 \xi)^3$$

$\alpha_0 = a_0$ — α_0 - DIMENSIONLESS

$\alpha_1 = a_1$ — α_1 - LENGTH

$\alpha_2^2 = a_2$ — α_2 - LENGTH

$\alpha_2^3 = a_3$ — $\vdots$

$$H(\xi) = e^{+i\pi w(\xi)} = e^{+i\pi\left(\alpha_0 + \alpha_1 \xi + (\alpha_2 \xi)^2 + \dots\right)}$$

$$= \underbrace{e^{+i\pi\alpha_0}}_{\text{CONSTANT}} \; \underbrace{e^{i\pi\alpha_1\xi}}_{\text{LINEAR}} \; \underbrace{e^{+i\pi(\alpha_2\xi)^2}}_{\text{QUADRATIC}} \; \underbrace{e^{+i\pi(\alpha_3\xi)^3}}_{\text{CUBIC}} \dots$$

FILTER THM

$$h(x) = \mathcal{F}_1^{-1}\{H(\xi)\} = \mathcal{F}_1^{-1}\{e^{+i\pi\alpha_0}\} * \mathcal{F}_1^{-1}\{e^{+i\pi\alpha_1\xi}\} * \dots$$

(ZERO-ORDER) CONSTANT PHASE IMPULSE RESPONSE

$$\mathcal{F}_1^{-1}\{e^{+i\pi\alpha_0}\, 1(\xi)\} = e^{+i\pi\alpha_0}\, \mathcal{F}_1^{-1}\{1(\xi)\}$$

$$= e^{+i\pi\alpha_0}\, \delta(x)$$ (INFINITESIMAL SUPPORT)

$$g(x) = f(x) * e^{+i\pi\alpha_0}\, \delta(x)$$

$$= \underbrace{f(x) \cdot e^{+i\pi\alpha_0}}_{\text{REDISTRIBUTE}} = f(x) \cdot \cos(\pi\alpha_0) + i\, f(x) \cdot \sin(\pi\alpha_0)$$

REDISTRIBUTE AMPLITUDE AMONG REAL & IMAGINARY PARTS

ZERO - ORDER FILTER

$$|g(x)|^2 = |f(x)|^2 \Longrightarrow \int_{-\infty}^{+\infty} |g(x)|^2 \, dx = \int_{-\infty}^{+\infty} |f(x)|^2 \, dx$$

LINEAR - PHASE FILTER

$$\cancel{h_1(x)} \quad H_1[\xi] = e^{+i\pi\alpha_1\xi}\, 1(\xi) = e^{+2\pi i\left(\frac{\alpha_1}{2}\right)\xi} \cdot 1(\xi)$$

$$h_1(x) = \mathcal{F}_1^{-1}\{H_1(\xi)\} = \delta\left(x + \frac{\alpha_1}{2}\right)$$

INFINITESIMAL SUPPORT

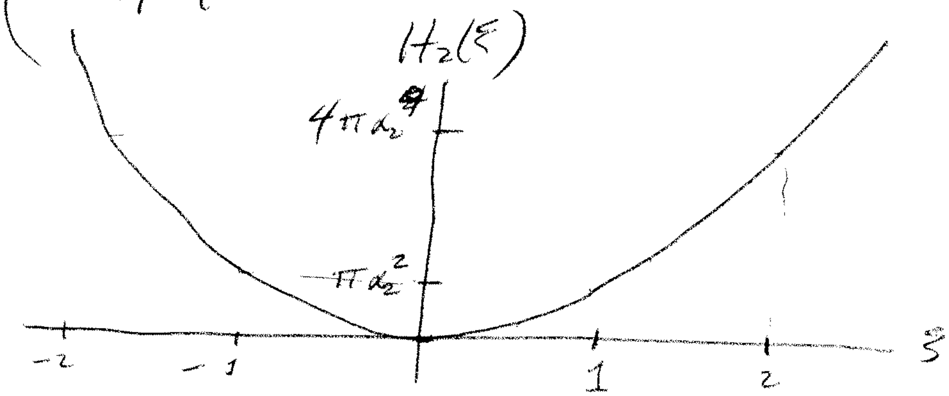
$$g(x) = f(x) * \delta\left(x + \frac{\alpha_1}{2}\right) = f\left(x + \frac{\alpha_1}{2}\right)$$

PRESERVES "SHAPE" OF $f(x)$

QUADRATIC - PHASE FILTER

$$H_2(\xi) = e^{+i\pi(\alpha_2 \xi)^2} \longrightarrow \underbrace{h_2(x) = \frac{1}{|\alpha_2|} e^{+i\frac{\pi}{4}} e^{-i\pi\left(\frac{x}{\alpha_2}\right)^2}}_{\text{INFINITE SUPPORT}}$$

$$\left\{ \begin{array}{l} \mathcal{F}_1^{-1}\{ e^{+i\pi \xi^2} \} = e^{+i\frac{\pi}{4}} e^{-i\pi x^2} \\ \mathcal{F}_1^{-1}\{ H(\alpha_2 \xi) \} = \frac{1}{|\alpha_2|} h\left(\frac{x}{\alpha_2}\right) \end{array} \right.$$



"SCRAMBLING" THE IMAGE

$$g(x) = f(x) * \frac{1}{|\alpha_2|} e^{+i\frac{\pi}{4}} e^{-i\pi\left(\frac{x}{\alpha_2}\right)^2}$$

IF $f(x) \propto (h(x))^*$, e.g. $f(x) = e^{+i\pi\left(\frac{x}{\alpha_2}\right)^2}$, THEN $g(x) \propto \delta(x)$

OUTPUT MAGNITUDE "RESEMBLES" INPUT MAGNITUDE

$\boxed{\text{FRESNEL DIFFRACTION}}$ = CORNU SPIRAL

ADDED LARGER PHASE INCREMENTS AT HIGHER FREQUENCIES

$$H_3(\xi) = e^{+i\pi(\alpha_3 \xi)^3} = \underbrace{\underbrace{\cos(\pi \alpha_3^3 \xi^3)}_{\text{even}} + i\, \underbrace{\sin(\pi \alpha_3^3 \xi^3)}_{\text{odd}}}_{\text{HERMITIAN}}$$

$h_3(x)$ IS REAL

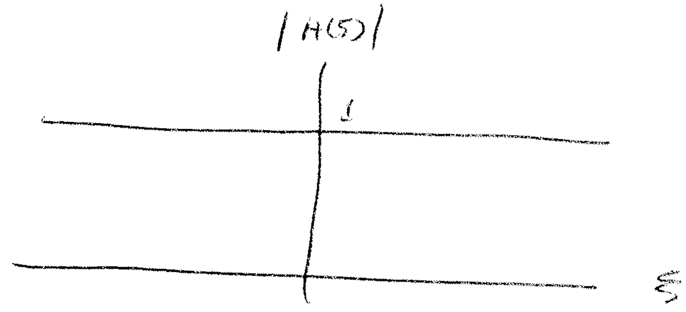
$\mathcal{F}_1^{-1} \downarrow$ even, REAL

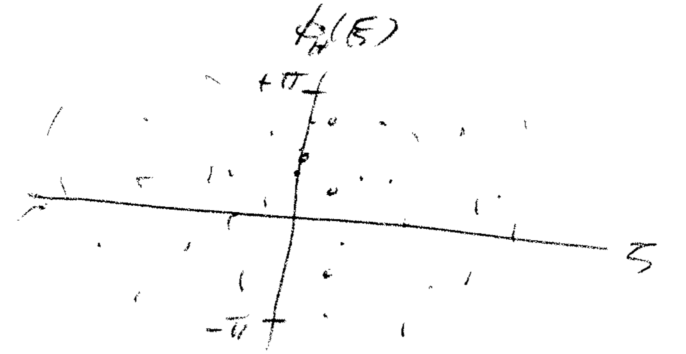
$\mathcal{F}_1^{-1} \downarrow$ ODD, REAL

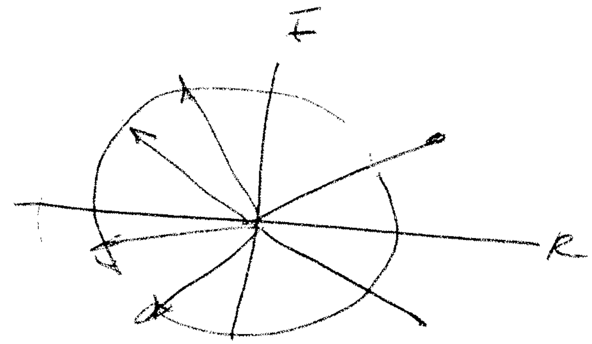
$$\underbrace{\text{even, REAL} \quad \text{ODD, REAL}}_{h_3(x) \text{ IS REAL}}$$

RANDOM - PHASE FILTER $\quad H[\xi] = \underline{\underline{1[\xi]}}\, e^{+i\pi N(\xi)}$

ADDING RANDOM NUMBER OF RADIANS AT EACH FREQ.

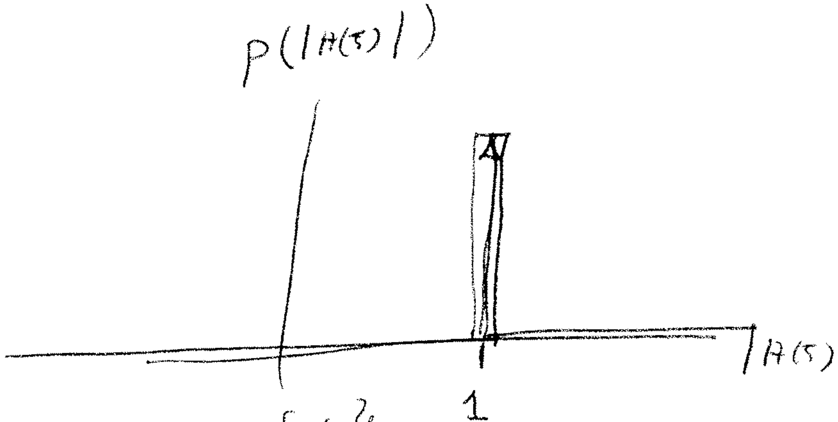


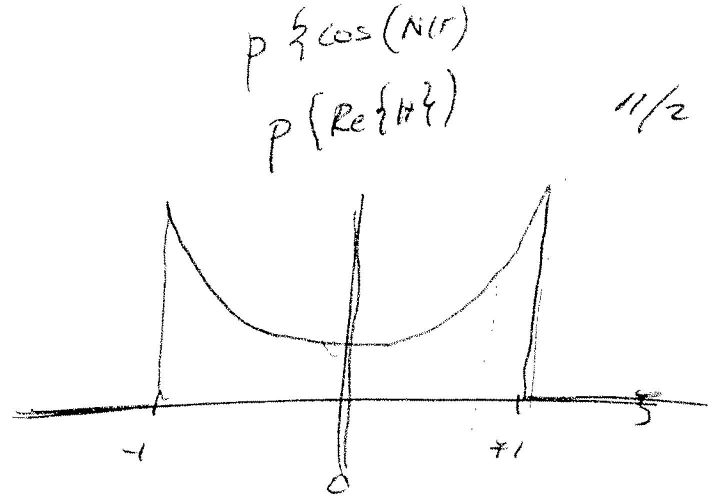


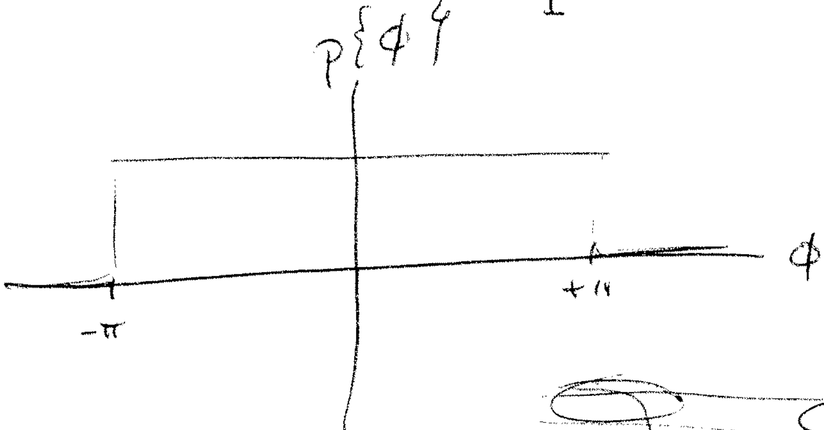


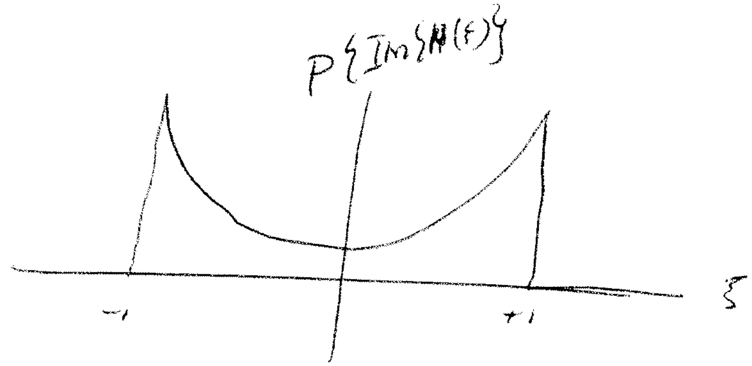
$$|H(\xi)| = 1(\xi) \Rightarrow h(x) \star h(x) = \delta(x)$$

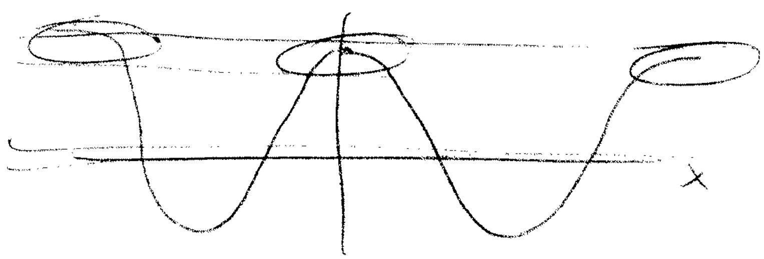
11/2 - 9











$$h(x) = \mathcal{F}_1^{-1}\left\{ e^{+i\pi N(\xi)} \right\}$$

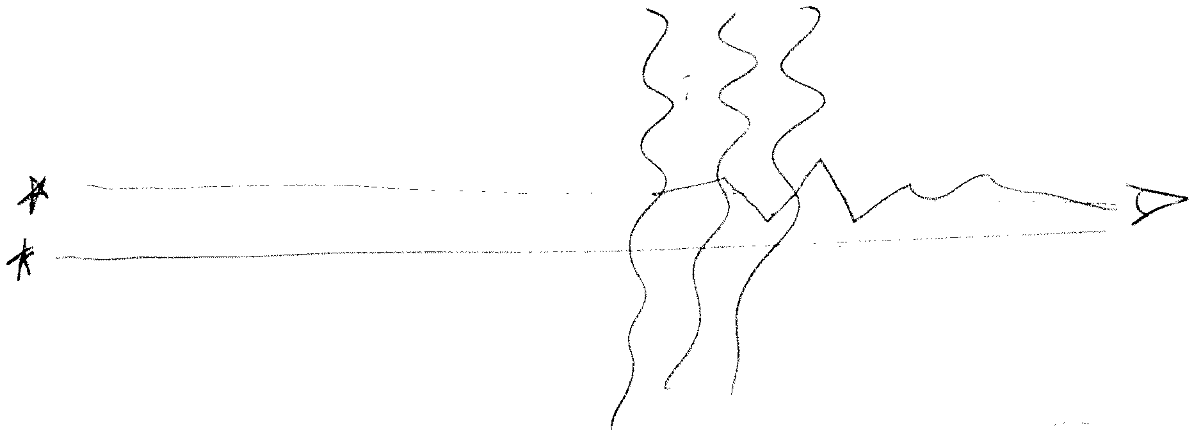
$$-1 \quad -\cancel{1} \le N(\xi) < \cancel{1}\ 1$$

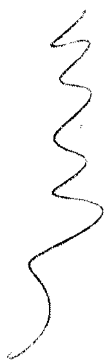
PROBABILITY DISTRIBUTION OF $h(x)$ $\longrightarrow$ GAUSSIAN IN BOTH REAL/IMAG
(HISTOGRAM)

CONSEQUENCE OF CENTRAL LIMIT THEOREM § 13.3

(1) ADD RANDOM NUMBERS FROM SAME DISTRIBUTION,
PROB. DIST. OF SUM TENDS TO BE GAUSSIAN

$$\mathcal{F}_1^{-1}\left\{ \cos(N(\xi)) \right\} = \int_{-\infty}^{+\infty} \cos(N(\xi))\, e^{+2\pi i \xi x}\, d\xi = f(x)$$





$$H(\xi) = I(\xi)\, e^{+i\pi N(\xi)}$$

STELLAR SPECKLE INTEROMETRY

1970 – LABEYRIE

IMAGING OF PHASE OBJECTS

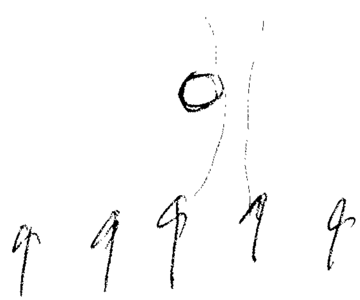
$$f(x) = 1(x)\, e^{+i\phi_f(x)}$$

MOST IMAGING SYSTEMS MEASURE $|g(x)|^2$

$$h(x) = \delta(x) \Rightarrow g(x) = f(x)$$

$$|g(x)|^2 = |f(x)|^2 = 1(x)$$

SCHLIEREN IMAGING



$$f(x) = 1(x)\, e^{i\phi_f(x)}$$

$$= \sum_{n=0}^{\infty} \frac{(i\phi_f(x))^n}{n!}$$

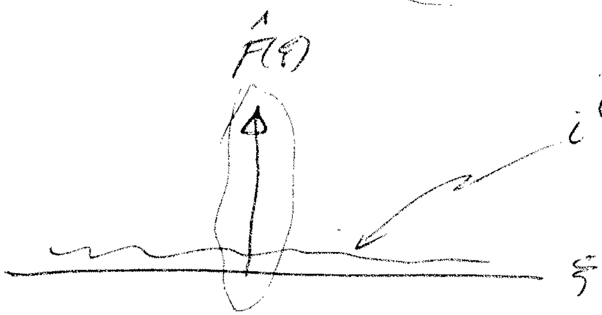
$$= 1 + i\phi_f(x) - \frac{(\phi_f(x))^2}{2} + \cdots$$

IF $|\phi_f(x)| \gtrsim 0$, $(\phi_f(x))^2 \ll \phi_f(x)$

$$f(x) \cong 1 + \underline{i\phi_f(x)}$$

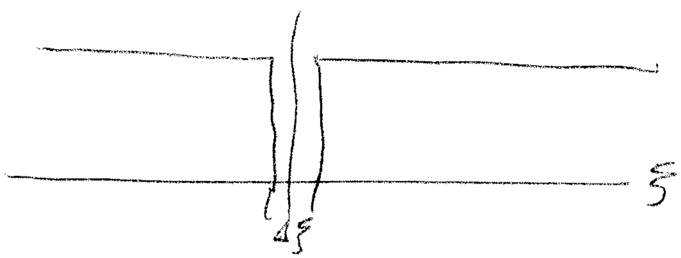
$$f(x) = e^{+i\phi_f(x)} \cong 1 + i\phi_f(x)$$

$$F(\xi) \cong \delta(\xi) + i\,\mathcal{F}_1\{\phi_f(x)\}$$



$H(\xi)$ IS HIGHPASS

$$H(\xi) = 1 - \mathrm{RECT}\left(\frac{\xi}{\Delta\xi}\right) \qquad 1 - \delta(\xi)$$



$$G(\xi) \cong F(\xi)\cdot H(\xi) \cong i\,\mathcal{F}_1\{\phi_f(x)\}$$

$$g(x) \cong i\,\phi_f(x) \Rightarrow |g(x)|^2 \simeq |\phi_f(x)|^2$$

IMPORTANCE OF $|F(\xi)|$ AND $\phi_F(\xi)$

$$|F(\xi)|\, e^{+i\phi_F(\xi)}$$