30 OCTOBER 2009

FILTERING

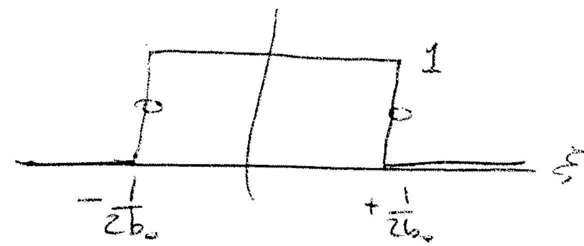
LOWPASS $\Rightarrow$ ATTENUATED/BLOCKED HIGH-FREQ. SINUSOIDS

HIGHPASS $\rightarrow$ BLOCKED LOW-FREQ. SINUSOIDS

IDEAL LPF & HPF $\Rightarrow$ LPF LOCAL ~~AVE~~ WEIGHTED SUM

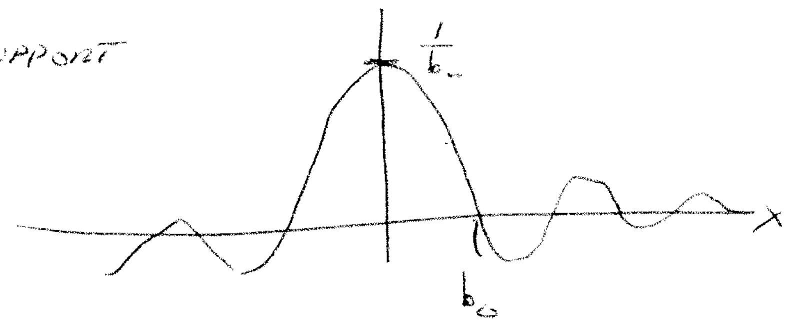
HPF LOCAL DIFFERENCE AT ORIGIN OF $h(x)$

IDEAL LPF $\quad H[\xi] = \mathrm{RECT}[b_0 \xi]$

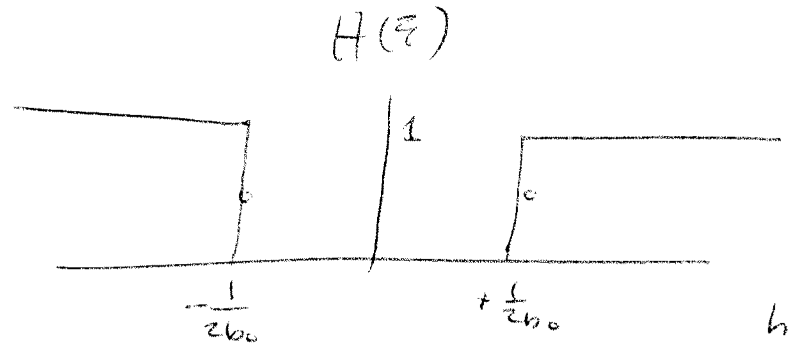


$$h(x) = \frac{1}{b_0} \mathrm{SINC}\left(\frac{x}{b_0}\right)$$

INFINITE SUPPORT

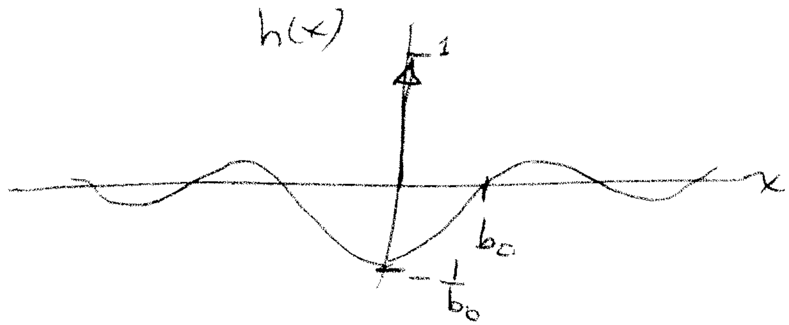


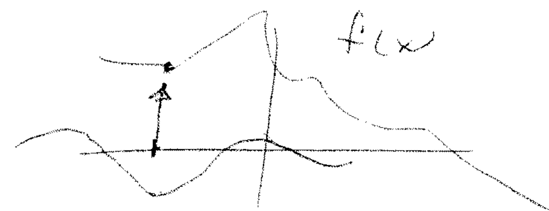
HPF



$$\boxed{1(\xi) - \mathrm{Rect}(b_0 \xi)}$$

$$h(x) = \delta(x) - \frac{1}{b_0}\,\mathrm{Sinc}\left(\frac{x}{b_0}\right)$$

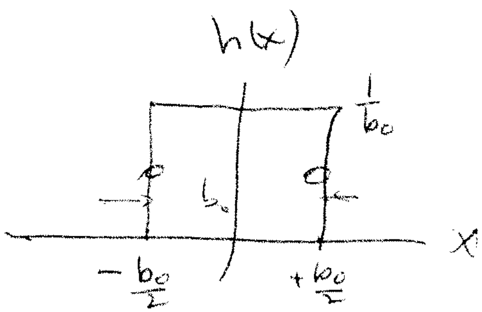


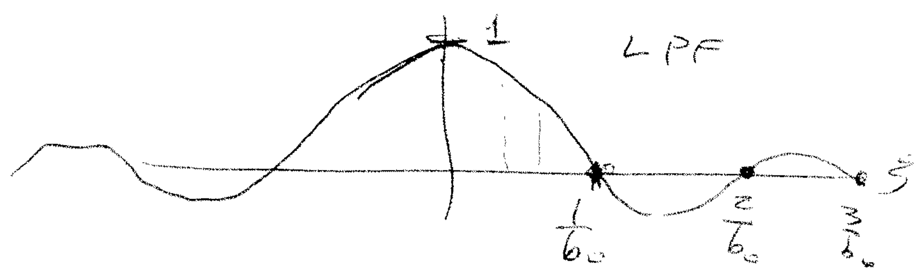


UNIFORM AVERAGER

$$h(x) = \frac{1}{b_0}\,\mathrm{Rect}\left(\frac{x}{b_0}\right) \quad ; \quad \int_{-\infty}^{+\infty} h(x)\,dx = 1$$

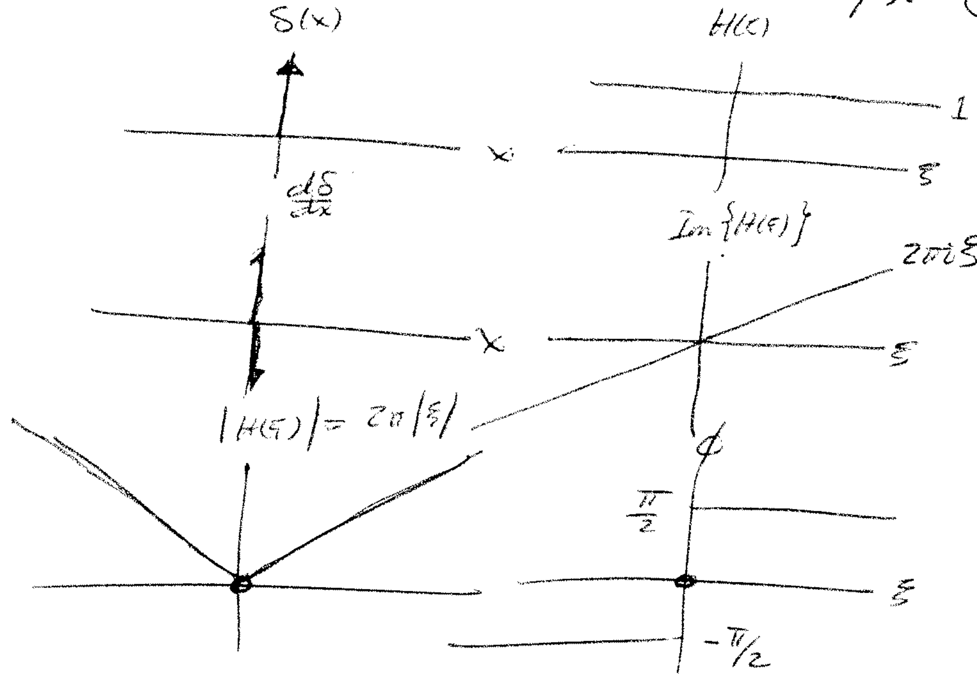
$$H(\xi) = \mathrm{Sinc}(b_0 \xi)$$





DIFFERENTIATORS

$$h(x) = \frac{d^n}{dx^n}\delta(x)$$



HIGHBOOST FILTER

(DC BLOCK)

$$H(\xi) = (2\pi i \xi)^n$$

DC ≡ DIRECT CURRENT (CONSTANT)

AC = ALTERNATING CURRENT (OSCILLATING)

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HIGH-BOOST FILTERS ⟹ (IMAGE) SHARPENERS

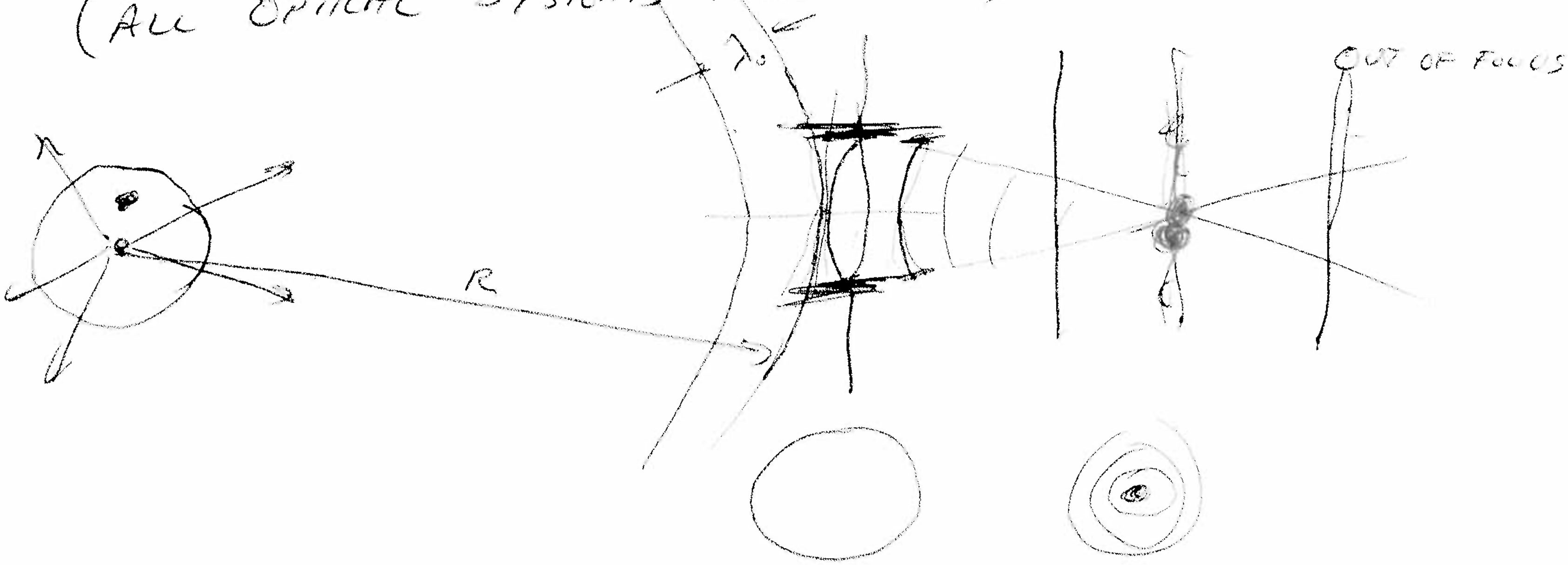
PASS LOW ~~FREQ~~ SINUSOIDS, AMPLIFY HIGH-FREQ

BLURRED IMAGE AS INPUT

↑

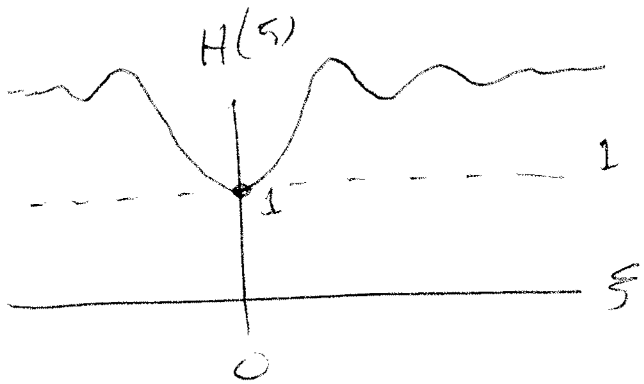
LPF (HIGH FREQUENCIES ATTENUATED)

(ALL OPTICAL SYSTEMS ARE LPF) ⟹ FINITE APERTURE



HIGH BOOST FILTER = SHARPOVEN

$$\left.\begin{array}{l} H[0] = 1 \\ H[\xi > 0] > 1 \end{array}\right\} \quad H_{HB}[\xi] = 1[\xi] + \underbrace{H_{HP}[\xi]}_{(1[\xi] - H_{LP}[\xi])}$$



$$= 2 \cdot 1(\xi) - H_{LP}[\xi]$$

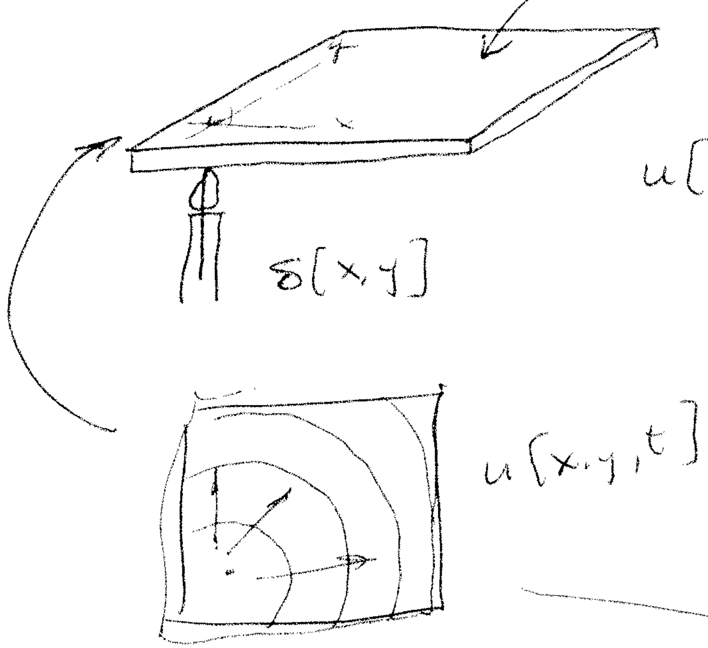
$$h_{HB}[x] = 2 \cdot \delta(x) - h_{LP}(x]$$

$$g(x) = f(x) * h_{HB}(x)$$

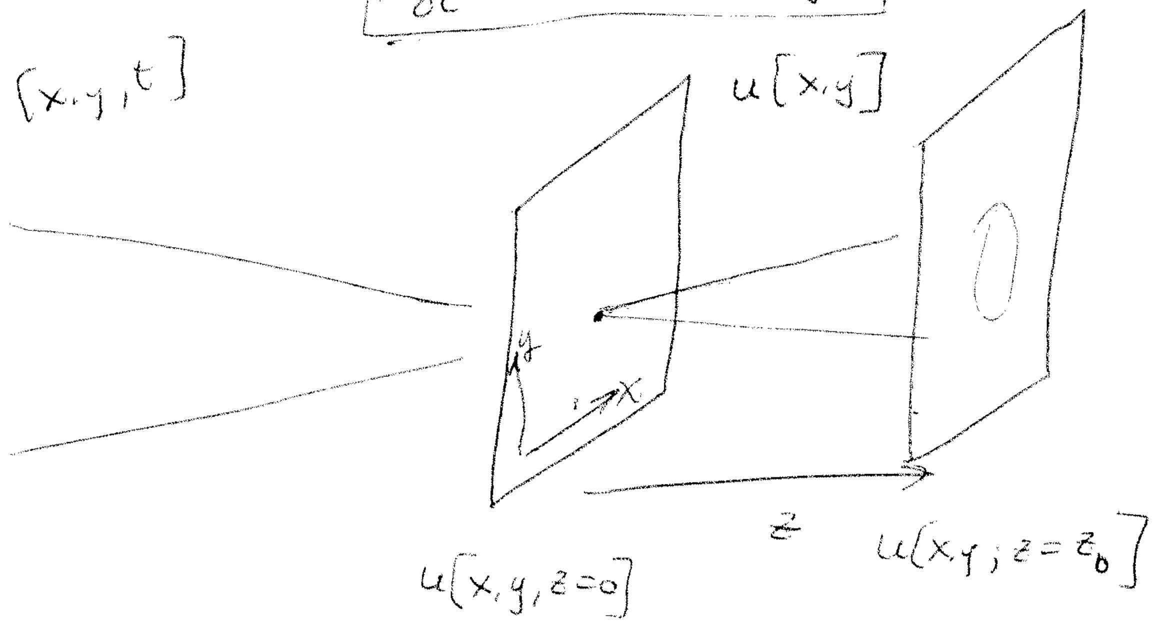
$$= 2 \cdot f(x) - f(x) * h_{LP}(x)$$

MODEL - BASED SHARPENING

HEAT DIFFUSION



$$\boxed{\frac{\partial u}{\partial t} \propto \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}}$$



LAPLACIAN OF u

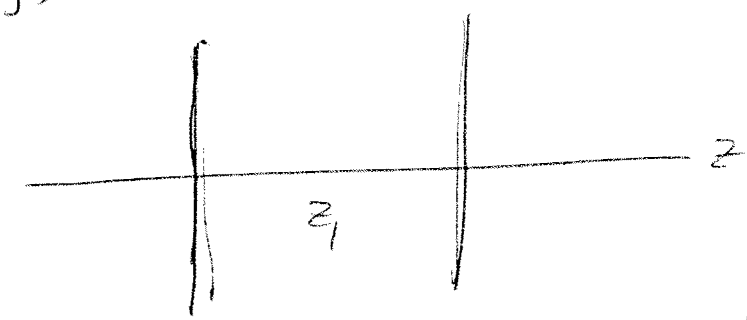
$$\frac{\partial u}{\partial t} = k \nabla^2 u \qquad \text{HEAT DIFFUSION}$$

$f[x,y,0]$ — IN FOCUS (SHARP) IMAGE

$f[x,y,z=z_1]$ — BLURRY IMAGE

$$\frac{\partial f}{\partial z} = k \nabla^2 f = k\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) f$$

$f[x,y,z=0]$ AS TAYLOR SERIES BASED ON $f[x,y,z=z_1]$



IF $z_1 \gtrsim 0$, $z_1^2 << z_1$

$$\underset{\text{IN FOCUS}}{f[x,y;z=0]} = \underset{\text{BLURRY}}{f[x,y;z=z_1]} + \frac{(0-z_1)}{1!}\frac{\partial f}{\partial z}\Big|_{z=z_1} + \cancel{\frac{(0-z_1)^2}{2!}\frac{\partial^2 f}{\partial z^2}\Big|_{z=z_1} + \cdots}$$

~~OUT~~

$$f[x,y;z=0] \cong f[x,y;z=z_1] - \frac{z_1}{1}\frac{\partial f}{\partial z}\Big|_{z=z_1}$$

$$= f(x,y;z=z_1) - (z_1 \cdot k)\underbrace{\nabla^2 f\Big|_{z=z_1}}_{\text{LAPLACIAN OF BLURRY IMAGE}}$$

"SHARP" IMAGE — BLURRY IMAGE

$$= f(x,y;z=z_1) * \delta(x,y) - \alpha\left(f[x,y;z_1] * \left(\delta''(x)\cdot\delta(y) + \delta(x)\delta''(y)\right)\right)$$

$$f[x,y;z=0] \cong f[x,y;z=z_1] * \underbrace{\left(\delta(x,y) - \alpha\left(\delta''(x)\delta(y) + \delta(x)\delta''(y)\right)\right)}_{h_{SHARPEN}(x)}$$

~~SH~~ IN FOCUS

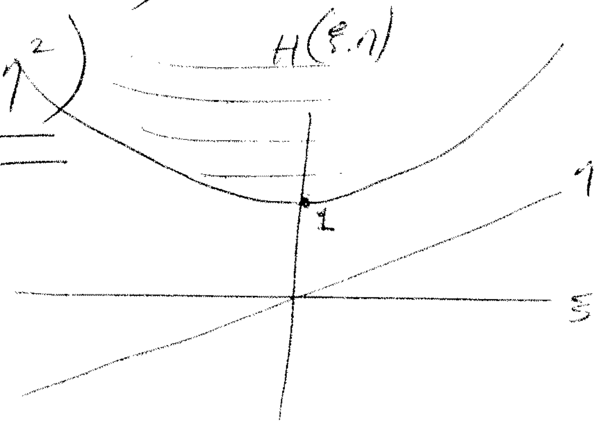
$$\delta(y)\,\delta''(x) \xrightarrow{\mathcal{F}_2} 1(\eta)(2\pi i\xi)^2 = -4\pi^2\xi^2(1(\eta))$$

$$+\ \delta(x)\,\delta''(y) \xrightarrow{\mathcal{F}_2} 1(\xi)\cdot(2\pi i\eta)^2 = \underline{-4\pi^2\eta^2(1(\xi,\eta))}$$

$$-4\pi^2(\xi^2+\eta^2) = -4\pi^2\rho^2$$

$$H_{HB}[\xi,\eta] = 1[\xi,\eta] - \alpha\left(-4\pi^2\rho^2\right)$$

$$= \underline{\underline{1[\xi,\eta]}} + 4\pi^2\alpha\,\underline{\underline{\left(\xi^2+\eta^2\right)}}$$



$$h_{HB}[x,y] = \delta(x,y) - \alpha\,\underline{\underline{\nabla^2}}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \alpha\left(\begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \alpha\left(\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}\right) = \boxed{\begin{bmatrix} 0 & -\alpha & 0 \\ -\alpha & 1+4\alpha & -\alpha \\ 0 & -\alpha & 0 \end{bmatrix}}$$

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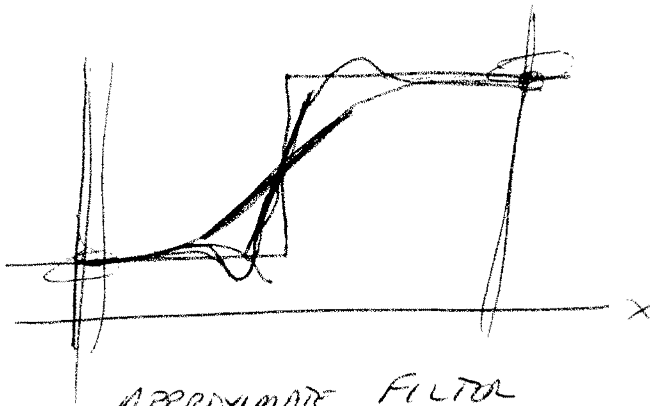
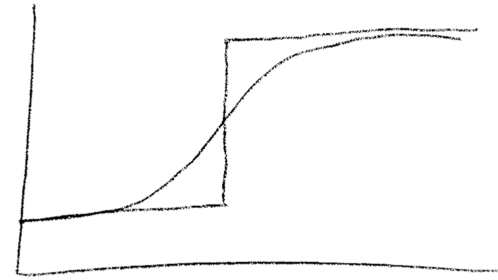
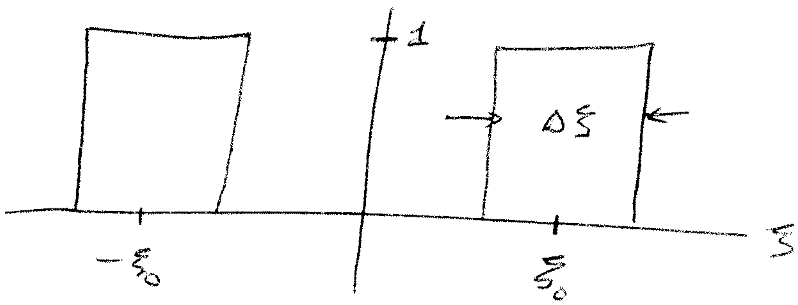


APPROXIMATE FILTER



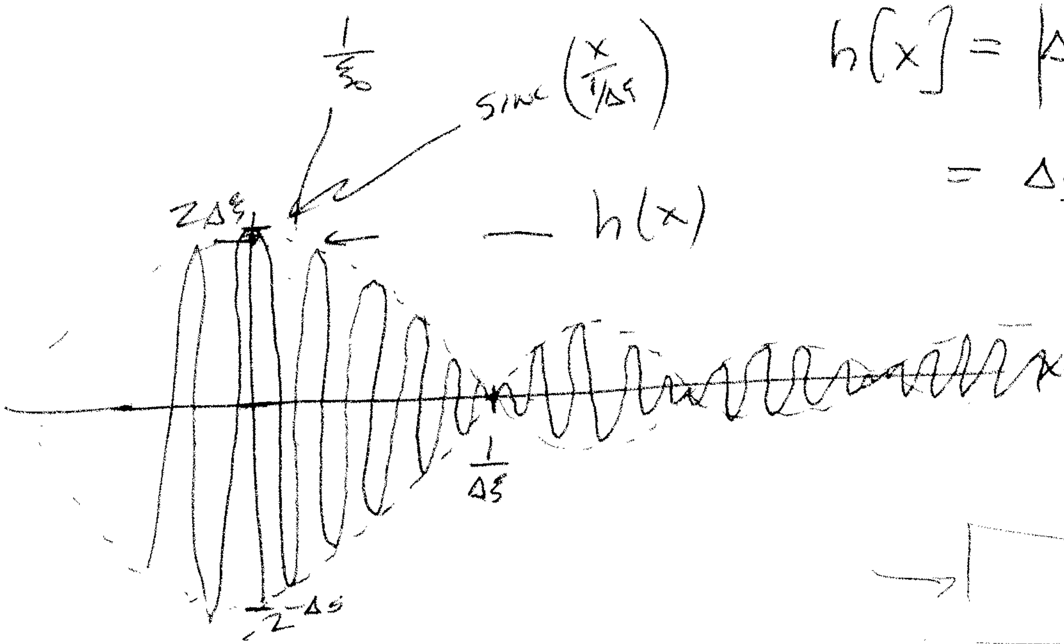
INVERSE FILTER

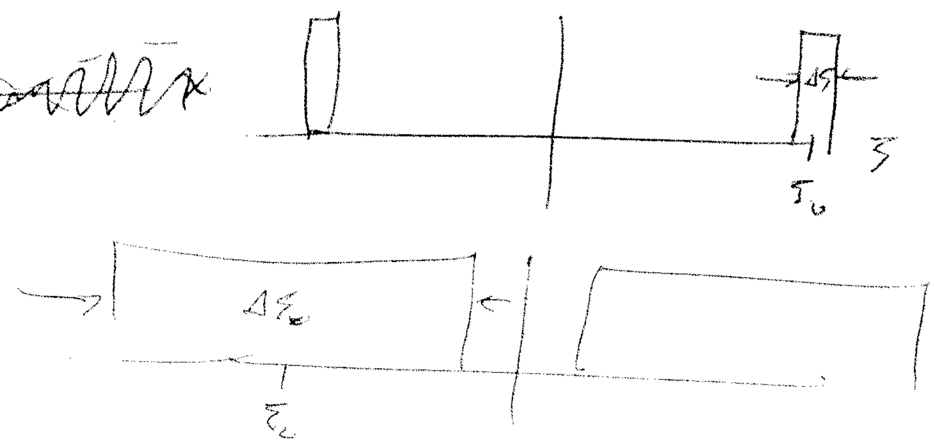
BANDPASS FILTERS — DIFFERENCE OF LOCAL AVERAGES



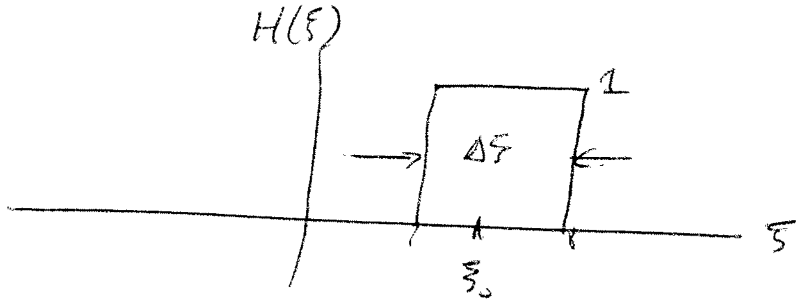
$$H[\xi] = \mathrm{RECT}\left(\frac{\xi+\xi_0}{\Delta\xi}\right) + \mathrm{RECT}\left(\frac{\xi-\xi_0}{\Delta\xi}\right)$$
$$= \mathrm{RECT}\left[\frac{\xi}{\Delta\xi}\right] * \left(\delta[\xi+\xi_0] + \delta(\xi-\xi_0)\right)$$

$$h[x] = |\Delta\xi| \cdot \mathrm{SINC}(\Delta\xi \cdot x) \cdot 2\cos(2\pi\xi_0 x)$$
$$= \Delta\xi \cdot \mathrm{SINC}\left[\frac{x}{1/\Delta\xi}\right] \cdot 2\cos(2\pi\xi_0 x)$$

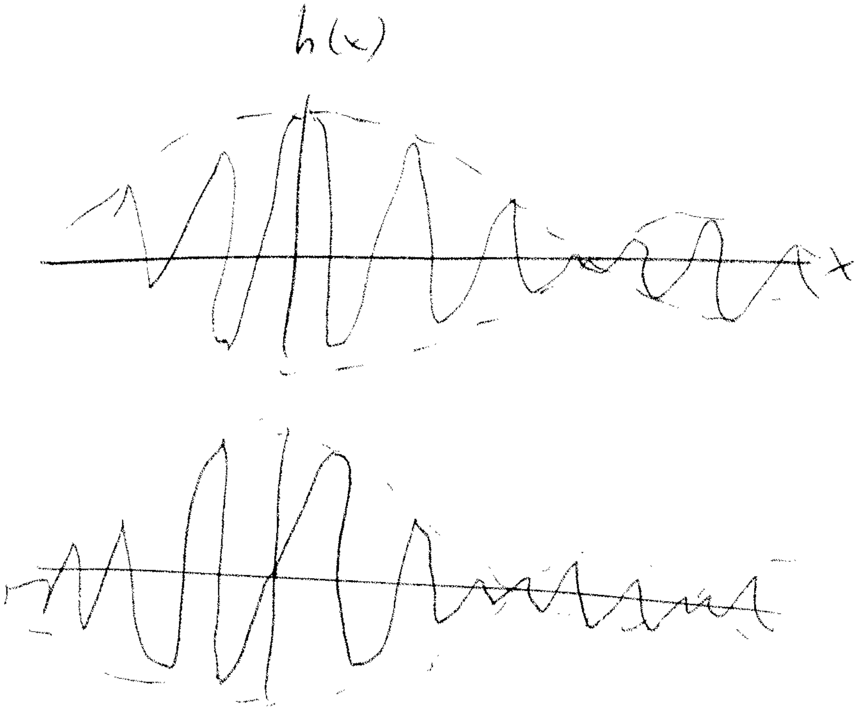




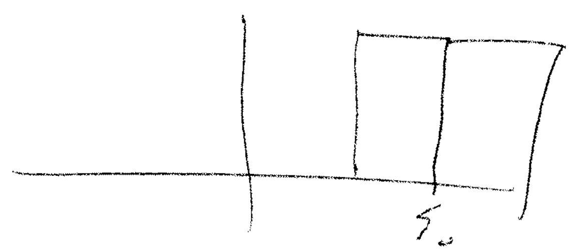
SINGLE - SIDEBAND BPF



$$\mathrm{Rect}\left(\frac{\xi}{\Delta\xi}\right) * \delta(\xi-\xi_0)$$

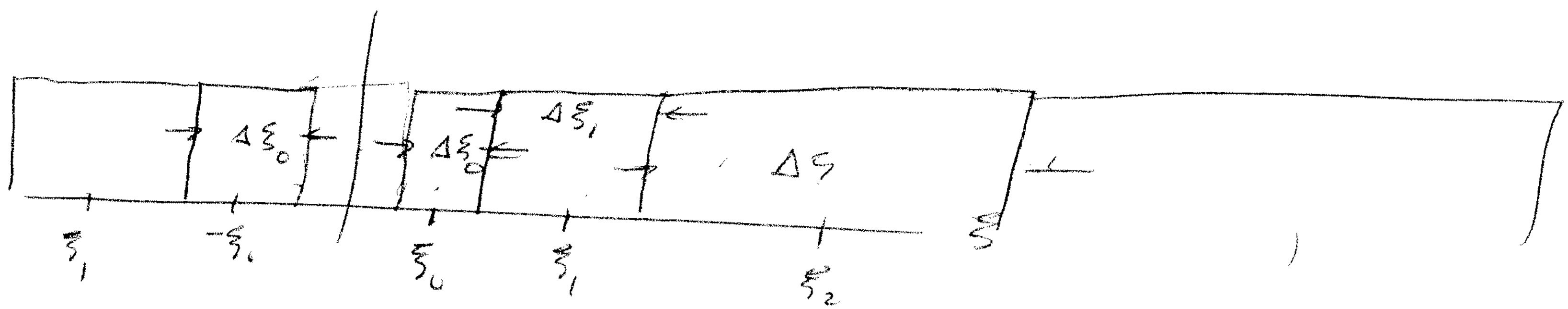


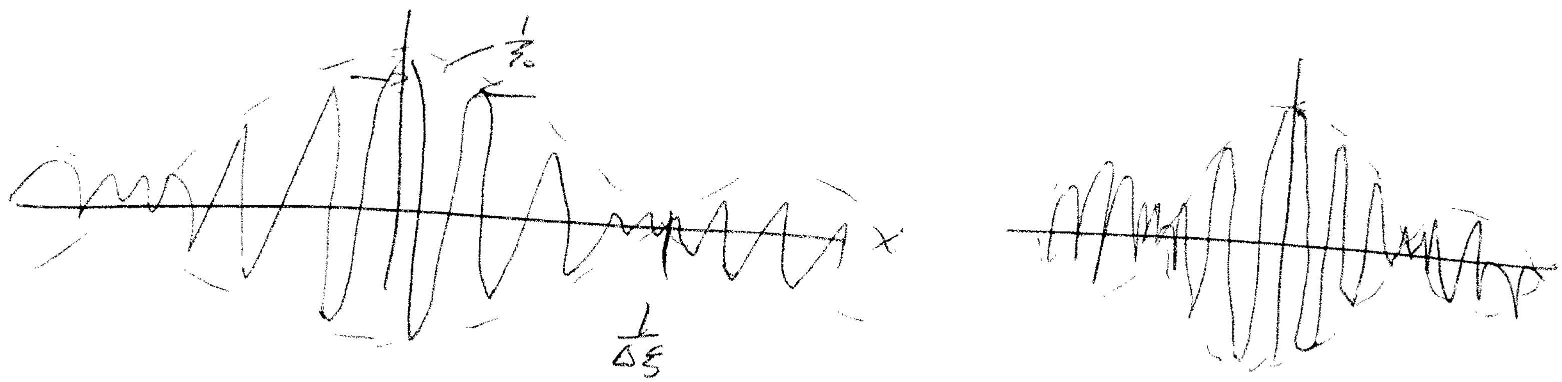
$$\Delta\xi\ \mathrm{sinc}(\Delta\xi\cdot x)\cdot\underbrace{e^{+2\pi i\xi_0 x}}_{\cos(2\pi\xi_0 x)\ +\ i\sin(2\pi\xi_0 x)}$$



WAVELET TRANSFORM

SERIES OF BANDPASS FILTERS SUCH THAT $\boxed{\dfrac{\Delta\xi}{\xi_0}}$ IS CONSTANT





PHASE FILTERS — ALLPASS FILTERS

$$H[\xi] = 1[\xi]\, e^{i\phi_H(\xi)}$$

$$|G(\xi)| = |F(\xi)| \cdot |H(\xi)| = |F(\xi)|$$

$$|G(\xi)|^2 = |F(\xi)|^2 \; ; \qquad \underbrace{|H(\xi)|^2 = 1(\xi)}$$

$$g(x) \star g(x) = f(x) \star f(x) \qquad h(x) \star h(x) = \delta(x)$$

AUTOCORRELATION IS PRESERVED

$$H(\xi) = e^{i\Phi_H(\xi)} = e^{i\pi W(\xi)} \qquad \pi \cdot W(\xi) = \Phi_H(\xi)$$

IF $W(\xi)$ IS "SMOOTH" $\Longrightarrow$ TAYLOR SERIES $W(\xi) = W(0) + \left.\dfrac{\partial W}{\partial \xi}\right|_{\xi=0} \cdot \xi + \left.\dfrac{\partial^2 W}{\partial \xi^2}\right|_{\xi=0} \xi^2 + \cdots$

10/90 (15)

$$H(\xi) = e^{i\pi W(\xi)} = \underbrace{e^{i\pi \cdot W(0)}}_{\text{CONSTANT PHASE}} \; \underbrace{e^{i\pi \frac{\partial W}{\partial \xi}\big|_{\xi=0} \xi}}_{\text{LINEAR PHASE}} \; \underbrace{e^{i\pi \frac{1}{2}\frac{\partial^2 W}{\partial \xi^2}\big|_{\xi=0} \xi^2}}_{\text{QUADRATIC PHASE}} + \cdots$$

$$h(x) = \mathcal{F}^{-1}\{H(\xi)\} = \underline{h_0(x)} * \underline{h_1(x)} * \underline{h_2(x)} * \cdots$$

CONSTANT

LINEAR

QUADRATIC

RANDOM