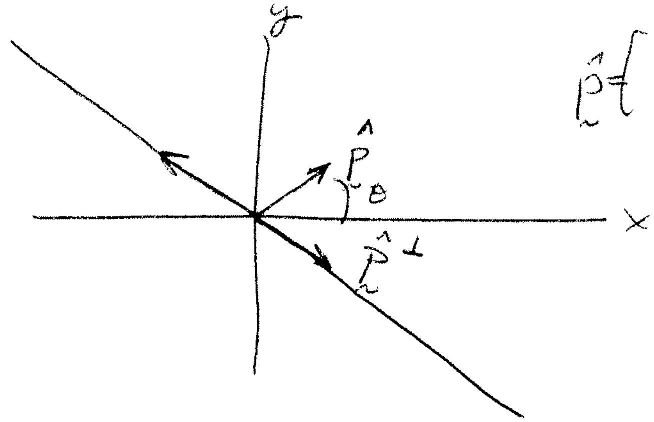
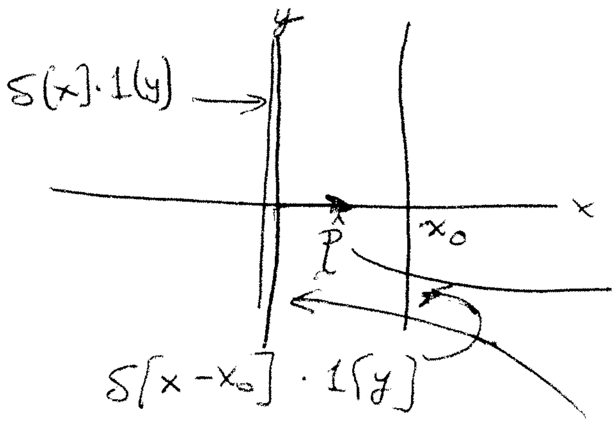
PROBLEM SESSION 10/9/2009

1

$\delta[ax + by]$



$\underset{\sim}{\hat{p}} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$



$\underset{\sim}{\hat{p}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

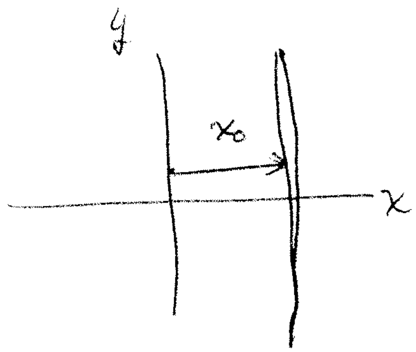
$\delta[\quad]\cdot 1[\quad]$

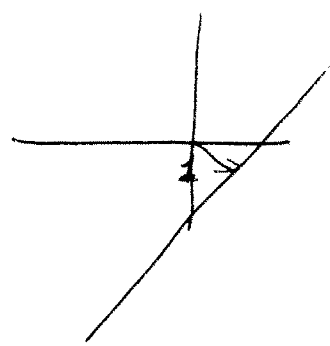
$\underset{\sim}{r} \circ \underset{\sim}{\hat{p}}$ $\qquad$ $\underset{\sim}{r} \circ \underset{\sim}{\hat{p}}^{\perp}$

$x\cos\theta + y\sin\theta$

$-x\sin\theta + y\cos\theta$

$x\sin\theta - y\cos\theta$



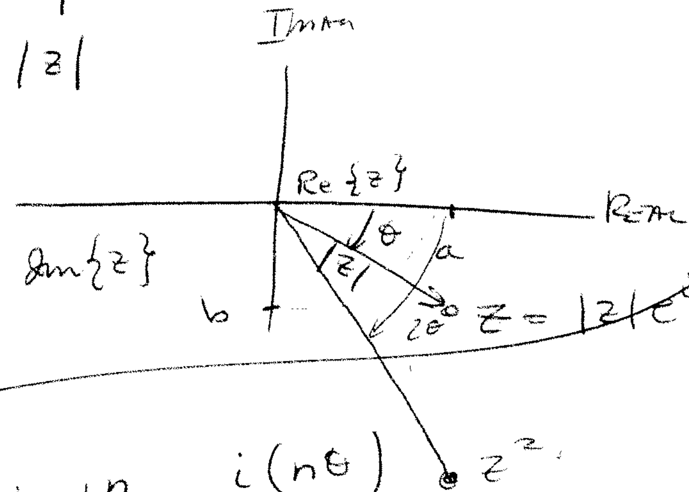


2

De Moivre's Theorem

$$z^n = \left(re^{i\theta}\right)^n \rightarrow \left(|z|e^{i\theta}\right)^n$$

r = $|z|$



$$\Rightarrow |z|^n e^{i(n\theta)}$$

e.g. $n=2 \Rightarrow z^2 = |z|^2 e^{i(2\theta)}$

③

$$z^{1/4} = |z|^{1/4}\, e^{i\frac{\theta}{4}}$$

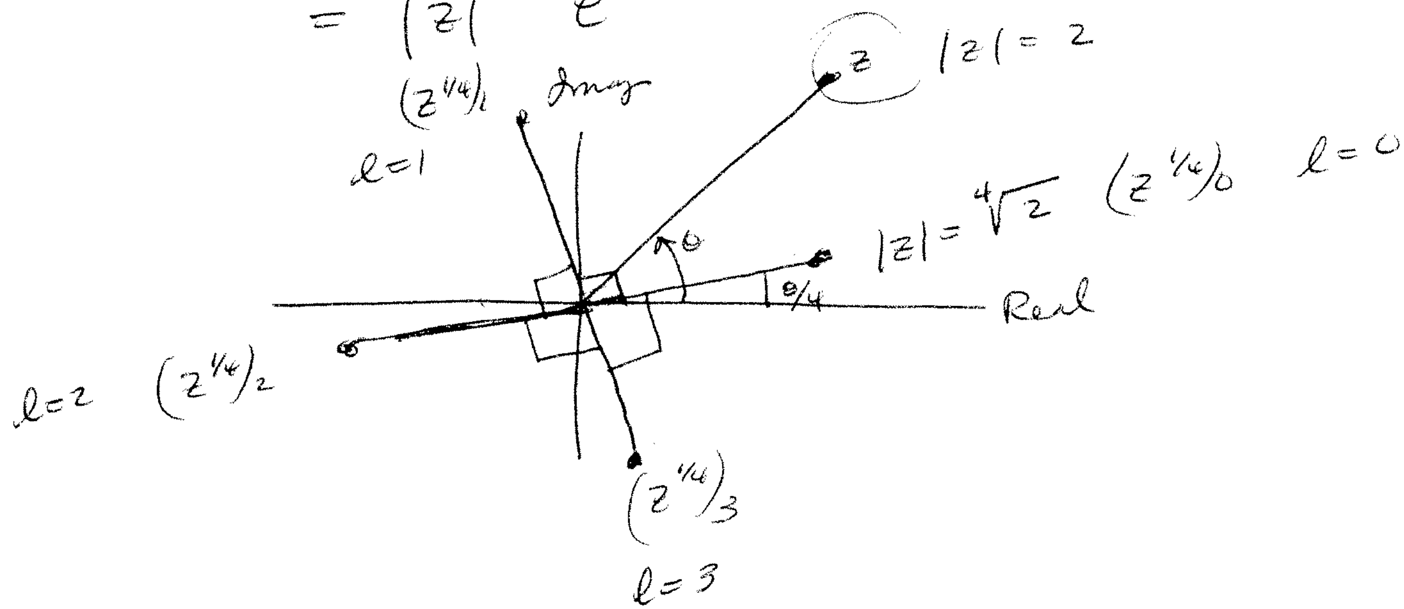
$a^4 = z$

$(-a)^4 = z$

$$\left(z^{1/4}\right)_l = |z|^{1/4}\, e^{i\left(\frac{\theta + 2\pi l}{4}\right)}$$

$l = 0, 1, 2, 3$

$$= |z|^{1/4}\, e^{i\left(\frac{\theta}{4} + \frac{\pi}{2} l\right)}$$

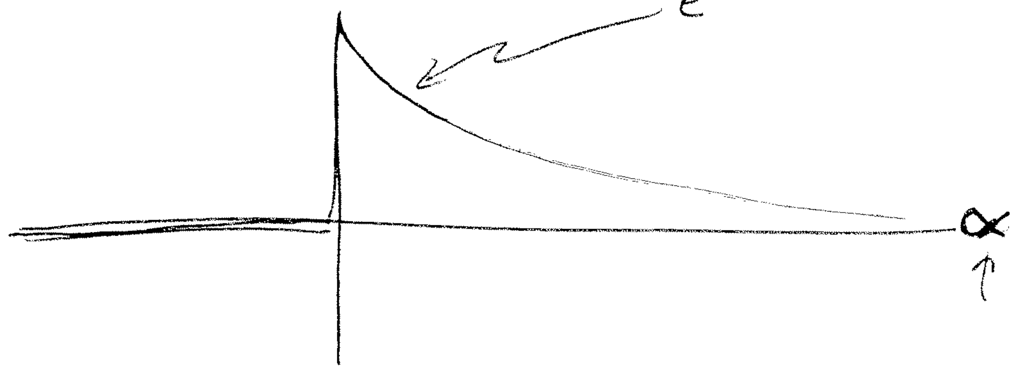


4

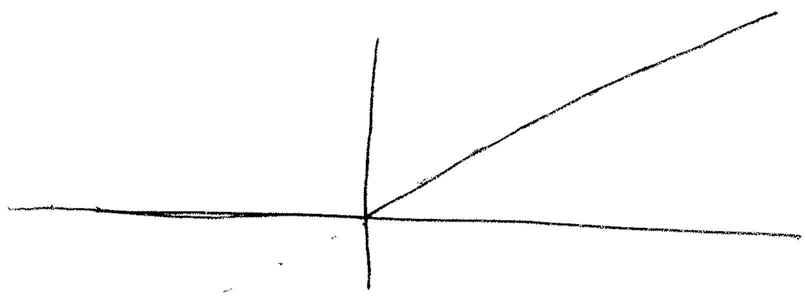
CONVOLUTION $f(x) * h(x) = h(x) * f(x)$

$$= \int_{-\infty}^{+\infty} f(\alpha)\, h(x-\alpha)\, d\alpha$$

$e^{-\alpha}\, \text{STEP}(\alpha)$



$\text{RAMP}(x) \equiv x\, \text{STEP}(x) = h(x)$

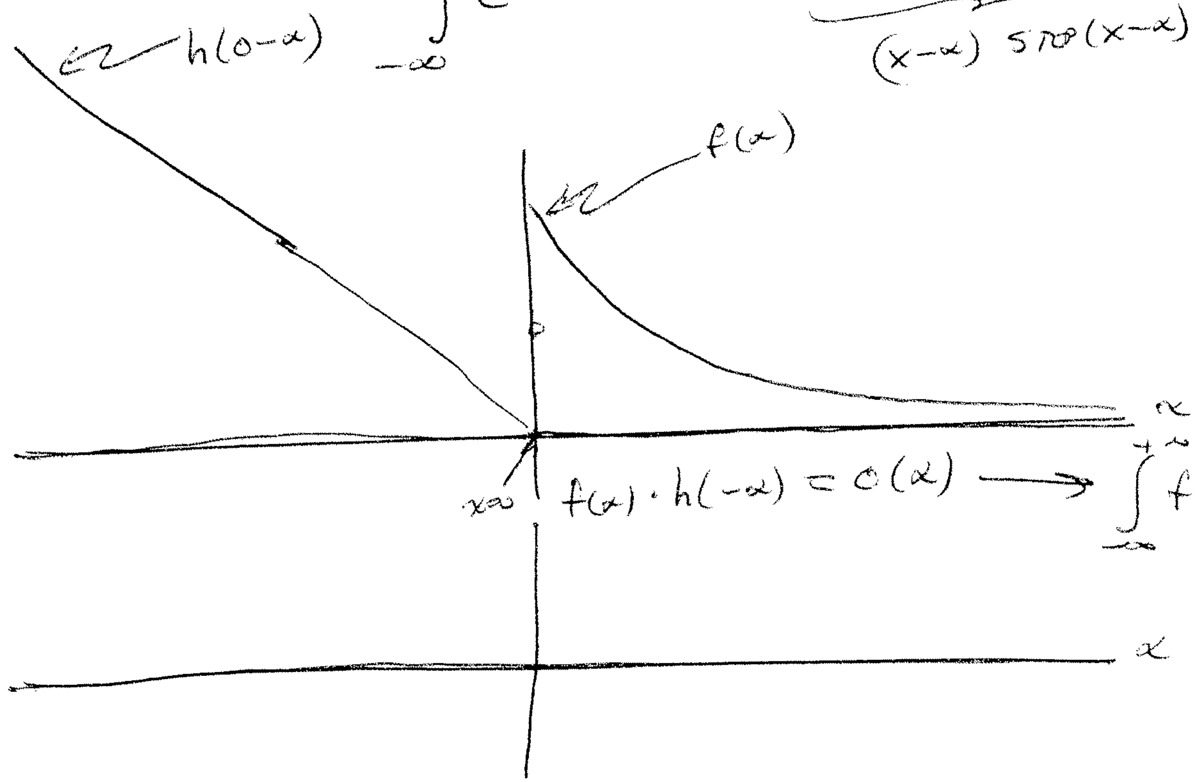


$$h(x-\alpha)\Big|_{x=0} = h(0-\alpha) = h(-\alpha)$$

$$e^{-x}\,\mathrm{STEP}(x) \ast \mathrm{RAMP}(x) = \int_{-\infty}^{+\infty} e^{-\alpha}\,\mathrm{STEP}(\alpha)\,\mathrm{RAMP}(x-\alpha)\,d\alpha$$ 5

$$@\ x=0 \Rightarrow \int_{-\infty}^{+\infty} e^{-\alpha}\,\mathrm{STEP}(\alpha)\,\mathrm{RAMP}(\underset{\uparrow\,0}{-\alpha})\,d\alpha$$

$$= \int_{-\infty}^{+\infty} e^{-\alpha}\,\mathrm{STEP}(\alpha)\,\underbrace{(-\alpha)\,\mathrm{STEP}(-\alpha)}_{(x-\alpha)\,\mathrm{STEP}(x-\alpha)\big|_{x=0}}\,d\alpha$$

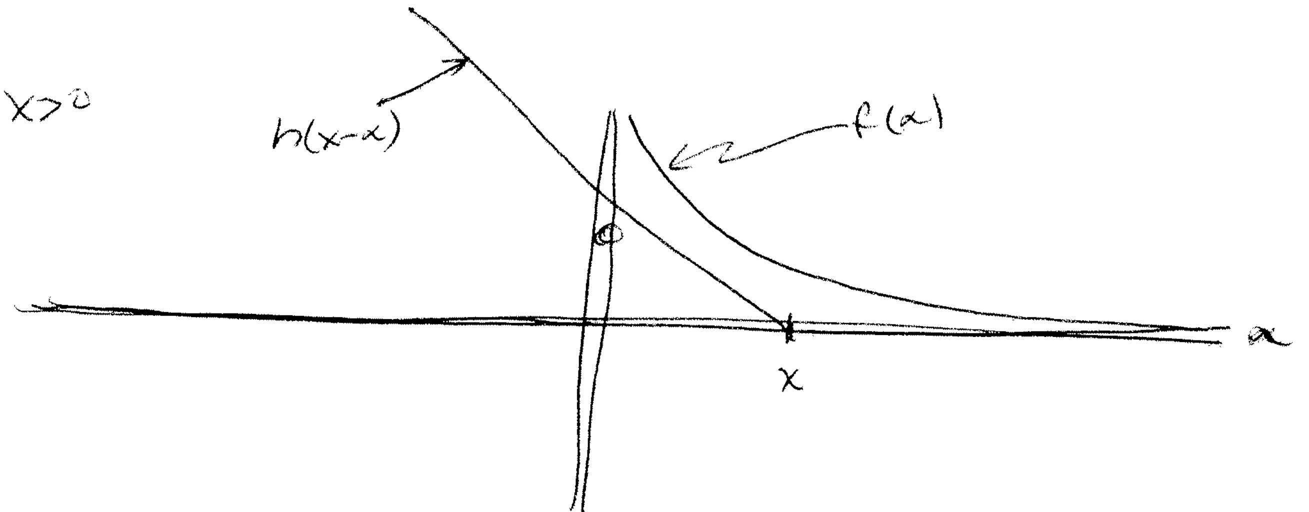


$$f(\alpha)\cdot h(-\alpha) = 0(\alpha) \longrightarrow \int_{-\infty}^{+\infty} f(\alpha)\,h(-\alpha)\,d\alpha = \cancel{g(x=0)}$$

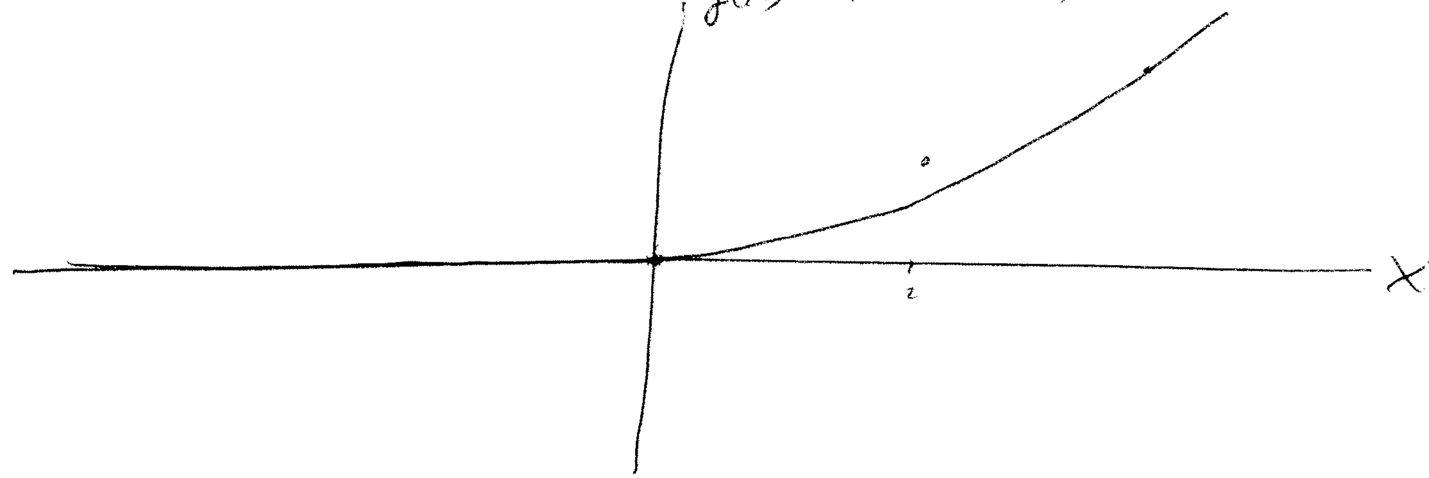
$$= g(x=0) = 0$$

6

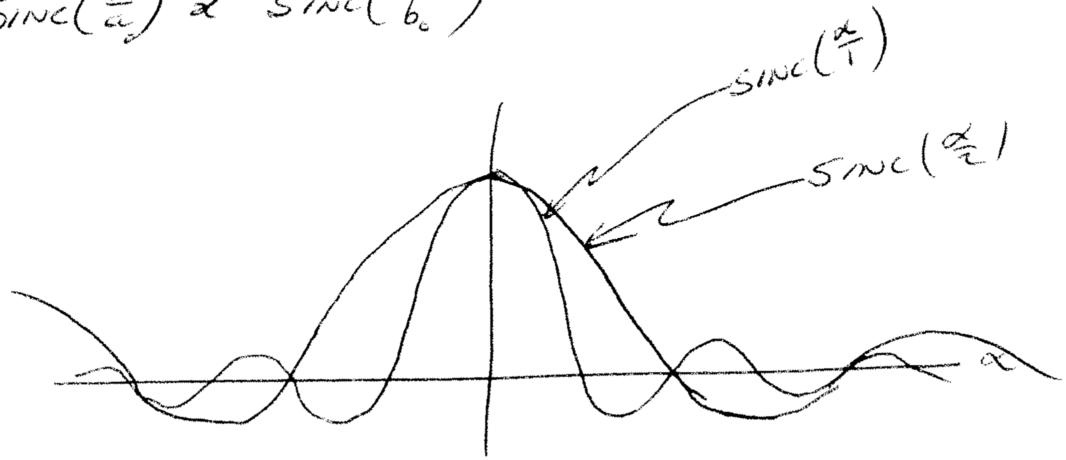
$x > 0$



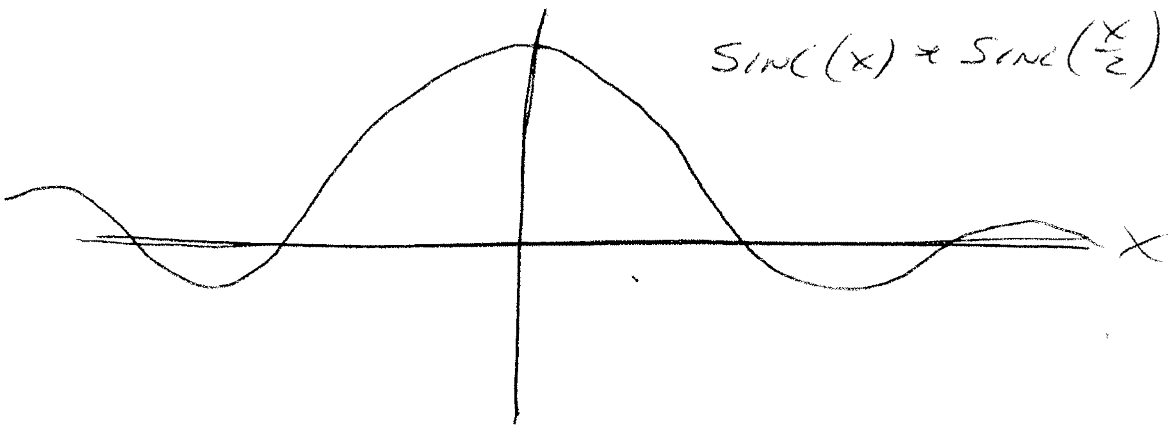
$g(x) = f(x) * h(x)$



$$\mathrm{sinc}\left(\frac{x}{a_0}\right) * \mathrm{sinc}\left(\frac{x}{b_0}\right)$$

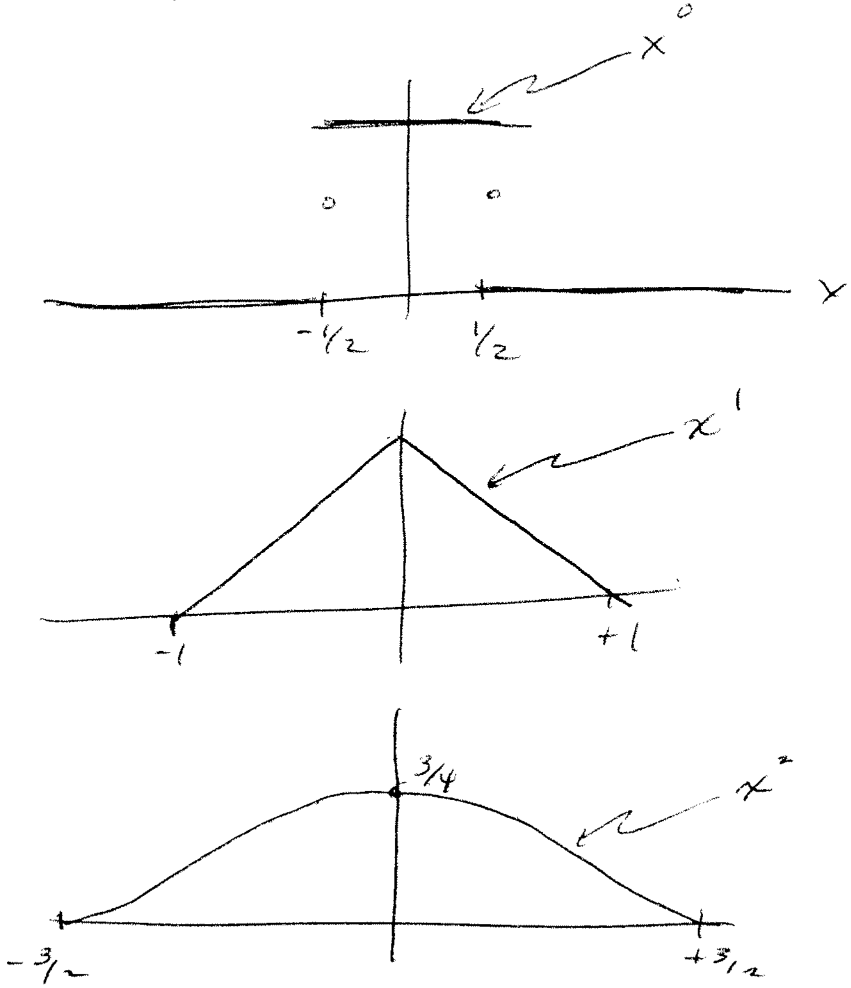


$$\mathrm{sinc}(x) * \mathrm{sinc}\left(\frac{x}{2}\right) = \mathrm{sinc}\left(\frac{x}{2}\right)$$



$f_1(x) * f_2(x) * f_3(x)$

$\mathrm{Rect}(x) * \mathrm{Tri}(x) = g(x)$



9

$$f(x) \ast h(x) = g(x)$$

$$\underline{e^{+i\pi x^2}} \ast \underline{e^{-i\pi x^2}} = \delta(x)$$

$$\boxed{e^{+i\pi\left(\frac{x}{2}\right)^2} \ast e^{-i\pi\left(\frac{x}{2}\right)^2} = |2|\,\delta\left(\frac{x}{2}\right) = 4\,\delta(x)}$$

$$\cancel{g\left(\frac{x}{2}\right)}$$

$$f\left(\frac{x}{2}\right) \ast h\left(\frac{x}{2}\right) = |2|\,g\left(\frac{x}{2}\right)$$

$$\cos(\pi x^2) \ast \cos(\pi x^2) = ?$$

$$\cos(\pi x^2)$$

$$\left[\cos(\pi x^2) + i\sin(\pi x^2)\right] \ast \left[\cos(\pi x^2) - i\sin(\pi x^2)\right]$$

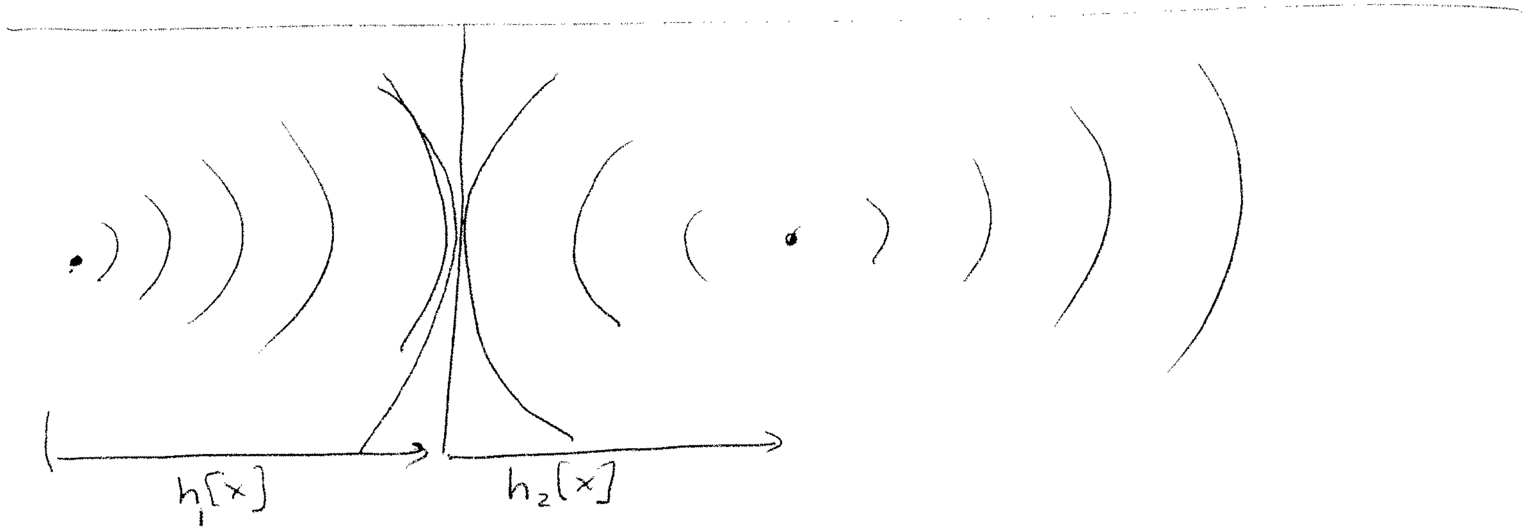
$$\cos(\pi x^2) \ast \cos(\pi x^2) + \cancel{i\sin(\pi x^2) \ast \cos(\pi x^2)} - \cancel{i\cos\pi x^2 \ast \sin(\pi x^2)}$$

$$+ \sin(\pi x^2) \ast \sin(\pi x^2)$$

$$e^{+i\pi x^2} \ast e^{-i\pi x^2} = \left[\cos(\pi x^2) \ast \cos(\pi x^2)\right] + \left[\sin(\pi x^2) \ast \sin(\pi x^2)\right]$$

$$= \delta(x)$$

$e^{+i\pi\left(\frac{x}{a_0}\right)^2} * e^{-i\pi\left(\frac{x}{b_0}\right)^2}$ HAS INFINITE SUPPORT



(11)

IF $f(x) * h(x) = g(x)$

THEN $F(\xi) \cdot H(\xi) = G(\xi)$

$$\hat{F}[\xi] = \frac{G[\xi]}{H[\xi]}$$

$$|a_0| \operatorname{SINC}\left(\frac{x}{b_0}\right) \quad \text{IF } b_0 > a_0$$

$$\uparrow$$

$$\operatorname{SINC}\left(\frac{x}{a_0}\right) * \operatorname{SINC}\left(\frac{x}{b_0}\right)$$

$$\downarrow$$

$$|a_0| \operatorname{RECT}\left(\frac{\xi}{1/a_0}\right) \cdot |b_0| \operatorname{RECT}\left(\frac{\xi}{1/b_0}\right) = |a_0||b_0| \cdot \operatorname{RECT}\left(\frac{\xi}{1/b_0}\right) \rightarrow |a_0||b_0| \operatorname{RECT}(\xi b_0)$$

IF $b_0 > a_0$

$$e^{-\pi\left(\frac{x}{3}\right)^2} * e^{-\pi\left(\frac{x}{4}\right)^2} = \frac{12}{5} e^{-\pi\left(\frac{x}{5}\right)^2}$$

$$\downarrow \qquad\qquad \uparrow$$

$$|3| e^{-\pi(3\xi)^2} \cdot |4| e^{-\pi(4\xi)^2} = |12| e^{-\pi(5\xi)^2}$$