2009-09-28 Monday

MAKEUP CLASS FRIDAY 10/2 4-6 PM EDT

#2 FRIDAY 10/23 4-6 PM (TENTATIVE)

THIS IS A TEST HELLO WORLD

1-D FUNCTIONS $\Rightarrow$ 2-D FUNCTIONS

REVIEW 1-D DELTA $\delta[x]$

(1) $\delta[x] = 0$ IF $x=0$

(2) $\displaystyle\int_{-\infty}^{+\infty} \delta(x)\, dx = 1$

$$\delta[x] = \int_{-\infty}^{+\infty} e^{+2\pi i \xi x}\, d\xi$$

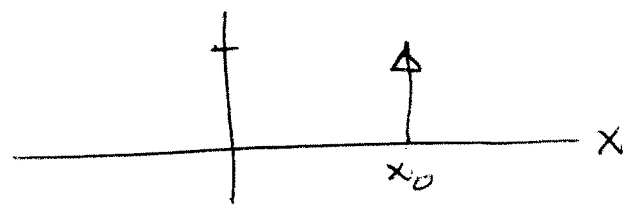
FOURIER SYNTHESIS

SIFTING

$$\boxed{f[x] \cdot \delta[x - x_0] = f[x_0]\,\delta(x - x_0)}$$

$$\int f(x)\, \delta[x - x_0]\, dx = f[x_0]$$

$$\underline{f[x - x_1]} \cdot \underline{\delta[x - x_0]}$$



$u \equiv x - x_1 \Rightarrow x = u + x_1$

$$f[u] \cdot \delta[(u + x_1) - x_0] = f(u]\ \delta[u - (x_0 - x_1))]$$

$$\int f(u)\,\delta\left[u-(x_0-x_1)\right] = \cancel{f[u]}$$

$$= f[x_0-x_1]\,\delta\left[u-(x_0-x_1)\right]$$

$$= f(x_0-x_1)\,\delta(x-\cancel{x_1}-x_0+\cancel{x_1})$$

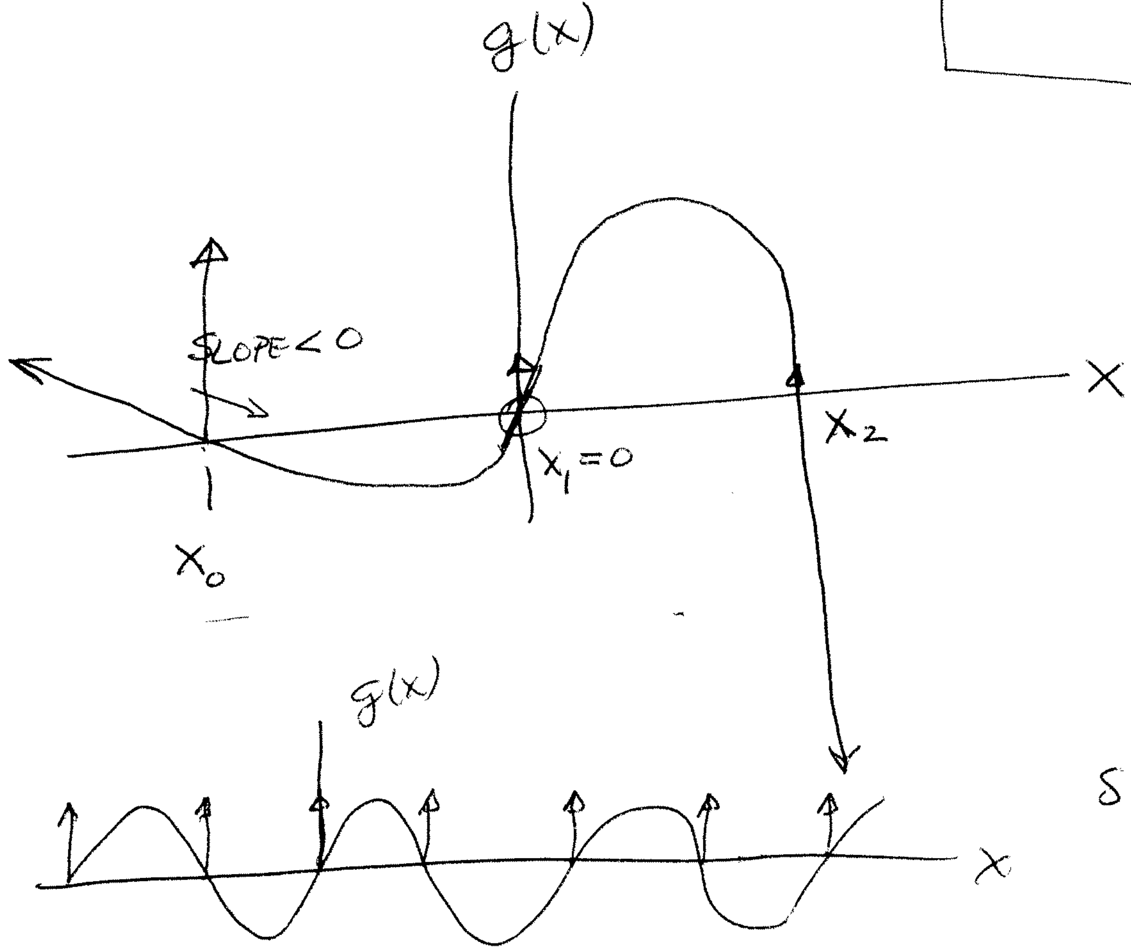
$$= f(x_0-x_1)\,\delta[x-x_0]$$

f EVALUATED AT x_0-x_1

$\delta[g(x)]$

IF $g(x_0) = 0$

$$\boxed{\delta(g(x)) = \frac{\delta(x - x_0)}{\left|\frac{dg}{dx}\right|_{x_0}}}$$



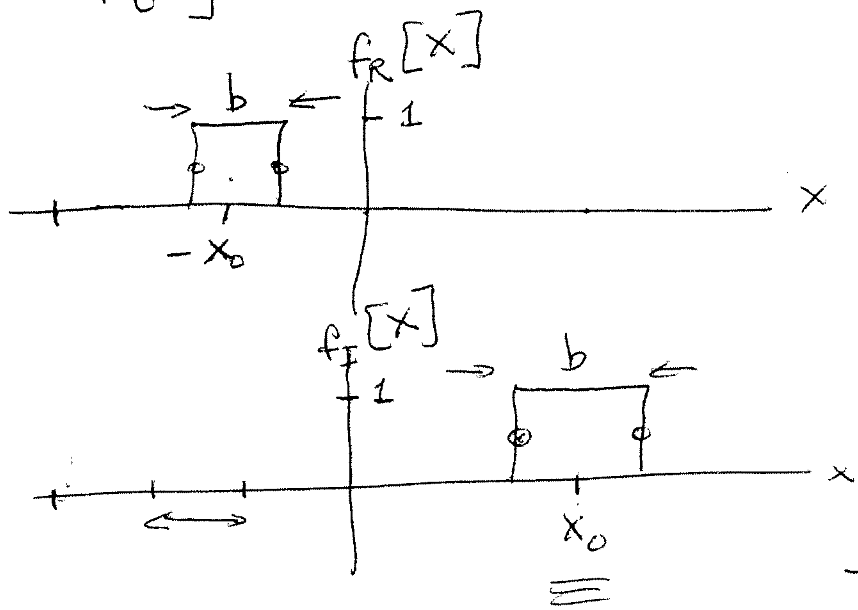
$\delta(g(x))$

$$\delta(\sin(2\pi\xi_0 x)) \propto \mathrm{comb}(\xi_0 x) = \mathrm{comb}\left(\frac{x}{1/\xi_0}\right)$$

PERIOD

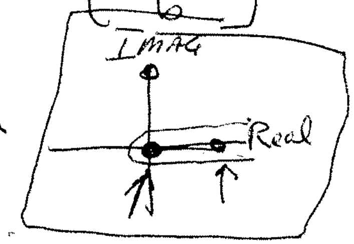
Complex-Valued 1-D Functions

$$f[x] = f_R[x] + i\, f_I[x]$$

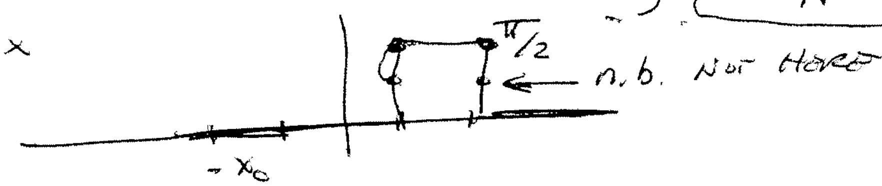


$$f[x] = \text{RECT}\left[\frac{x+x_0}{b}\right] + i\,\text{RECT}\left[\frac{x-x_0}{b}\right]$$

$$\left|f[x]\right| = \text{RECT}\left[\frac{x+x_0}{b}\right] + \text{RECT}\left[\frac{x-x_0}{b}\right]$$



$$\Phi\{f[x]\} = \text{TAN}^{-1}\left\{\frac{f_I[x]}{f_R[x]}\right\}$$

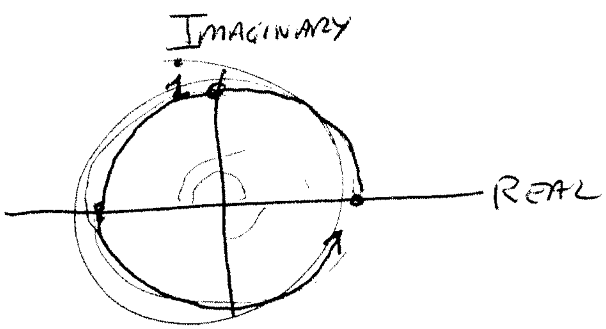


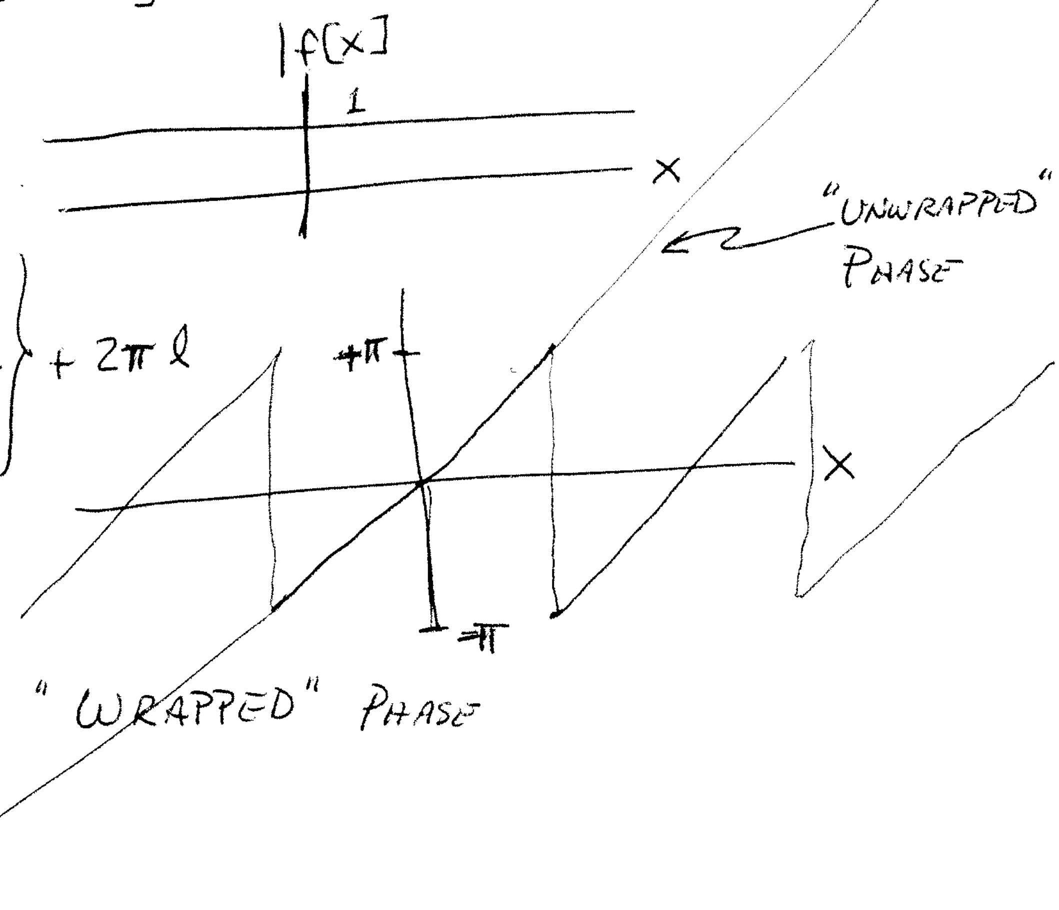
$$f[x] \cdot \text{SGN}[x]$$

$$f[x] = \underset{\uparrow \atop 1}{\cos}[2\pi\xi_0 x] + \underset{\uparrow \atop 1}{i} \sin[2\pi\xi_0 x] = e^{+2\pi}$$

$$|f[x]| = 1[x]$$

$$\Phi\{f(x)\} = \tan^{-1}\left\{\frac{f_I(x)}{f_R(x)}\right\} + 2\pi\ell$$





CEPSTRUM

$$f[x] = \cos[2\pi\xi_0 x] + i\sin[2\pi\xi_0 x] = 1[x]\, e^{+2\pi i\xi_0 x}$$

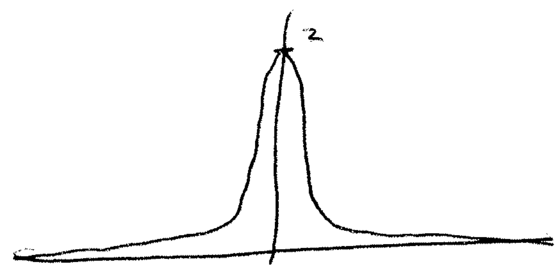
$$|f[x]| = 1(x)$$

$$\Phi\{f(x)\} = 2\pi\xi_0 x$$

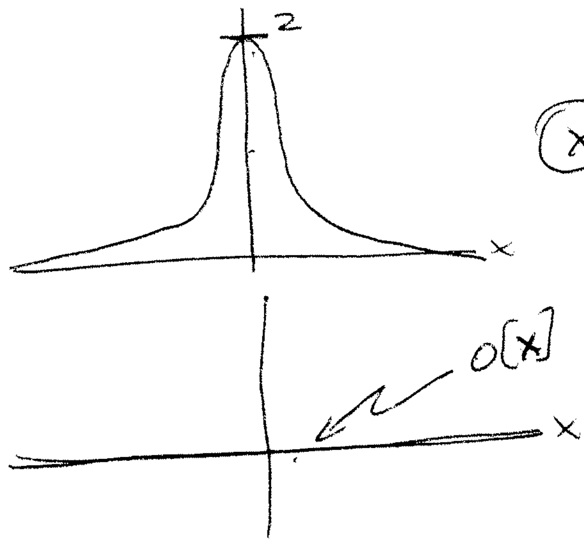
$$f[x] = 1[x]\, e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2} \qquad \Phi\{f(x)\} = \pi\left(\frac{x}{\alpha_0}\right)^2$$

n

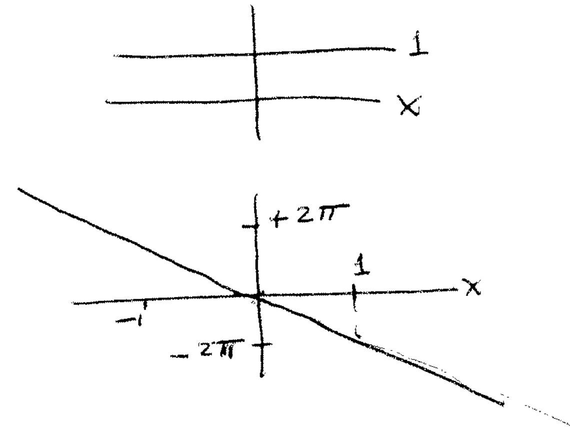
$$LOR[x] = \frac{2}{1+(2\pi x)^2}$$



$$\underline{\underline{LOR[x]}} \cdot \left(1 - i2\pi x\right)$$



x



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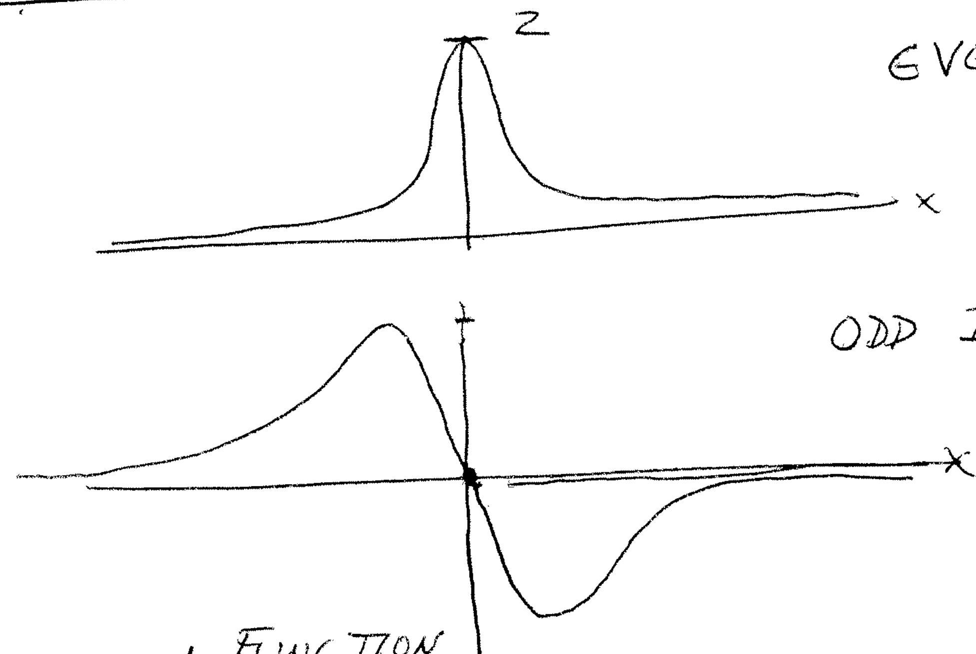


EVEN REAL

ODD IMAGINARY

HERMITIAN FUNCTION

$$\left| \frac{1-2\pi i x}{1+(2\pi x)^2} \cdot 2 \right| = \underset{\uparrow}{2} \left| \frac{1-2\pi i x}{1+(2\pi x)^2} \right|$$

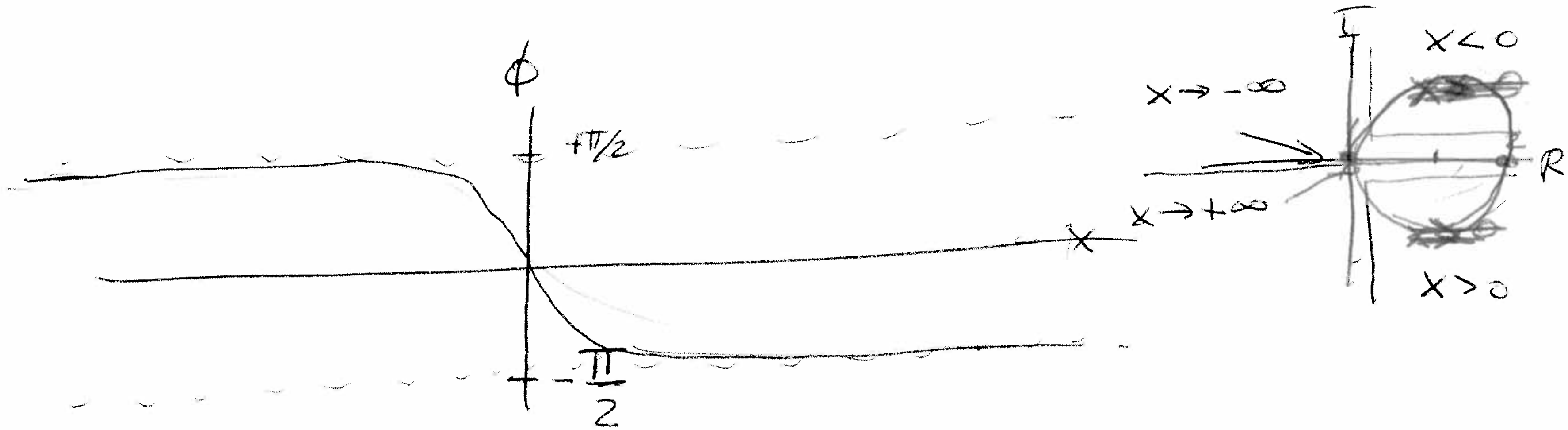
$$2 \cdot \sqrt{\left(\frac{1}{1+(2\pi x)^2}\right)^2 + \frac{(-2\pi x)^2}{\left(1+(2\pi x)^2\right)^2}} = \sqrt{\left(\frac{1}{1+(2\pi x)^2}\right)^2} \sqrt{\left(1+(2\pi x)^2\right)}$$

$$= 2 \cdot \frac{1}{1+(2\pi x)^2} \sqrt{1+(2\pi x)^2} = \sqrt{\frac{1}{1+(2\pi x)^2}} \cdot 2$$

$$\Phi = \tan^{-1}\left(\cancel{1-2\pi} \right.$$

$$\Phi = \tan^{-1}\left[\frac{\left(\dfrac{-2\pi x}{\cancel{1+(2\pi x)^2}}\right)}{\left(\dfrac{1}{\cancel{1+(2\pi x)^2}}\right)}\right] = \tan^{-1}\left[-2\pi x\right]$$

$$= -\tan\left[+2\pi x\right]$$



2-D Special Functions

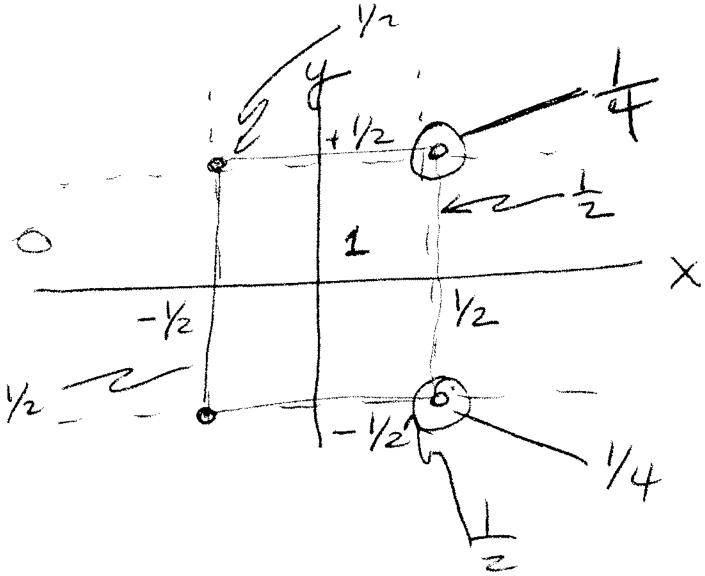
(1) Cartesian Separable Functions $f_1[x] \cdot f_2(y)$

(2) Polar Separable Functions $\underline{\underline{f_3(r)}} \cdot \underline{\underline{1(\theta)}}$
(Circularly Symmetric)

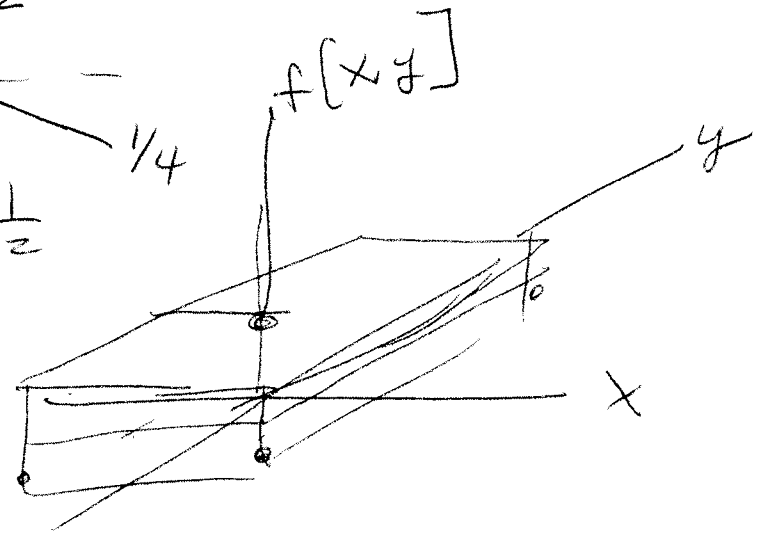
Volume $\displaystyle\iint_{-\infty}^{+\infty} f_1(x)\, f_2(y)\, dx\, dy$

$$= \int_{-\infty}^{+\infty} f_1(x)\, dx \cdot \int_{-\infty}^{+\infty} f_2(y)\, dy$$

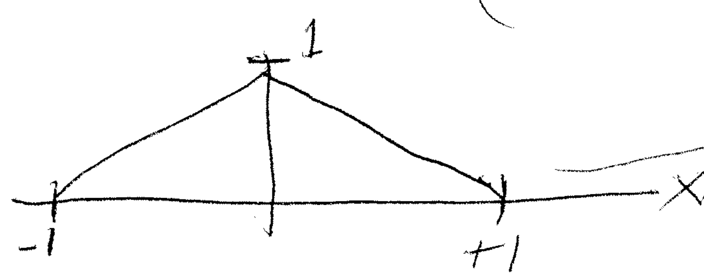
= Product of Areas

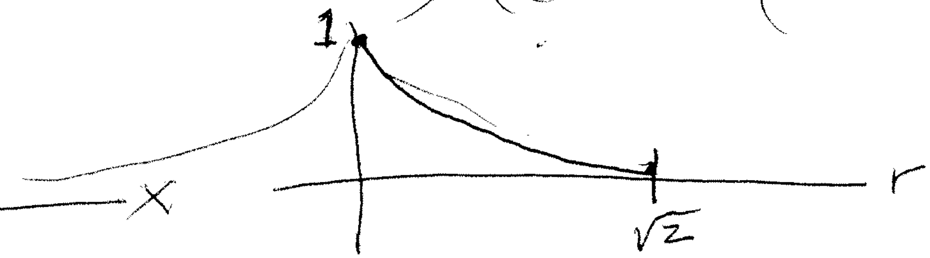


$\mathrm{RECT}[x]\cdot\mathrm{RECT}[y]$



$$TRI[x]\cdot TRI[y] = \left(\left(1-|x|\right)\cdot \mathrm{RECT}\left[\frac{x}{2}\right]\right)\left(\left(1-|y|\right)\cdot\left(\mathrm{RECT}\left(\frac{y}{2}\right)\right)\right)$$





2-D TRANSLATION $\quad f[x,y] \longrightarrow f[x-x_0,\ y-y_0]$ 13

2-D SCALING $\quad f[x,y] \longrightarrow f\left[\frac{x}{b}, \frac{y}{d}\right]$

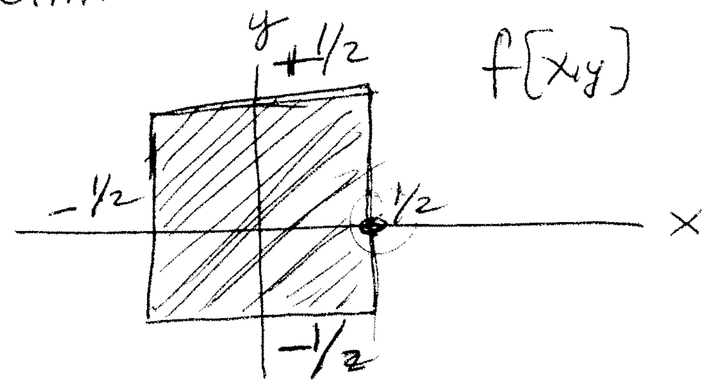
$$\iint_{-\infty}^{+\infty} \mathrm{TRI}[x,y]\, dx\, dy = 1 \cdot 1 = 1$$

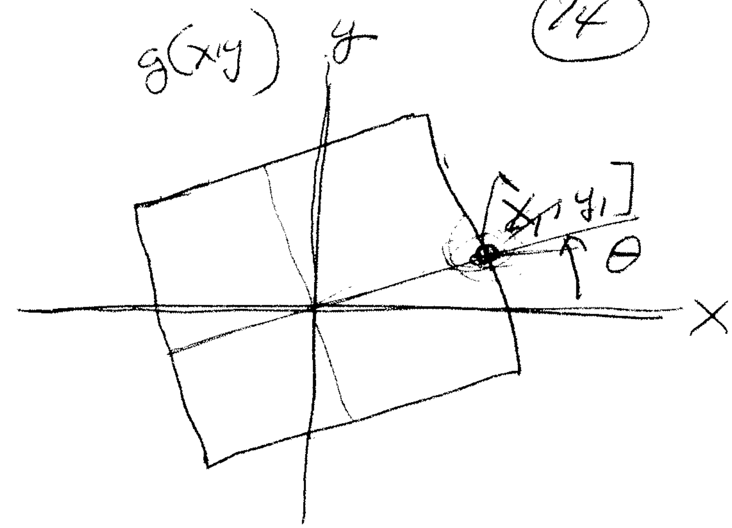
$$\mathrm{TRI}\left[\frac{x}{4}, \frac{y}{-2}\right] = \mathrm{TRI}\left[\frac{x}{4}\right] \cdot \mathrm{TRI}\left[\frac{y}{-2}\right]$$

$$= \mathrm{TRI}\left[\frac{x}{4}\right] \cdot \mathrm{TRI}\left[\frac{y}{+2}\right]$$

VOLUME $= 4 \cdot 2 = 8$

ROTATION OF 2-D FUNCTION





$$g[x,y] = f[x', y']$$

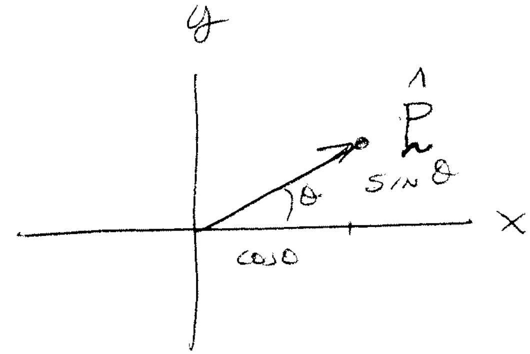
$$g[x_1, y_1] = f[x_1', y_1']$$

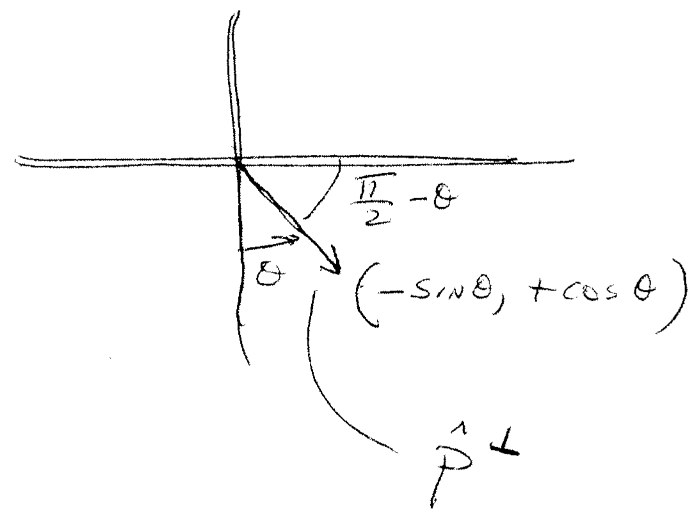
$$x_1' = x_1 \cos(\theta) + y_1 \sin(\theta)$$

$$y_1' = -x_1 \sin(\theta) + y_1 \cos(\theta)$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = x_1'$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \cdot \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = y_1'$$





$$\underset{\sim}{r} \cdot \hat{\underset{\sim}{p}} = x_1'$$

$$\underset{\sim}{r} \cdot \hat{\underset{\sim}{p}}^{\perp} = y_1'$$

$$\mathrm{RECT}[x]\cdot\mathrm{RECT}[y] \xrightarrow{R_\theta} \mathrm{RECT}[\underbrace{\underset{\sim}{r}\cdot\underset{\sim}{\hat{p}}}_{x\cos\theta + y\sin\theta}]\;\mathrm{RECT}[\underbrace{\underset{\sim}{r}\cdot\underset{\sim}{\hat{p}}^{\perp}}_{-x\sin\theta + y\cos\theta}]$$

EXAMPLES

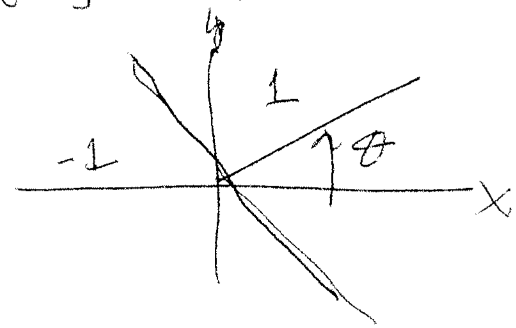
$$\mathrm{RECT}[x,y] = \mathrm{RECT}[x]\cdot\mathrm{RECT}[y]$$

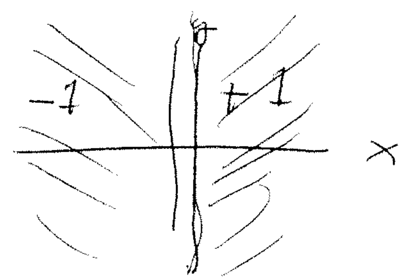
$$\mathrm{TRI}[x,y] = \mathrm{TRI}[x]\cdot\mathrm{TRI}[y]$$

$$1[x,y] = 1[x]\cdot 1[y]$$

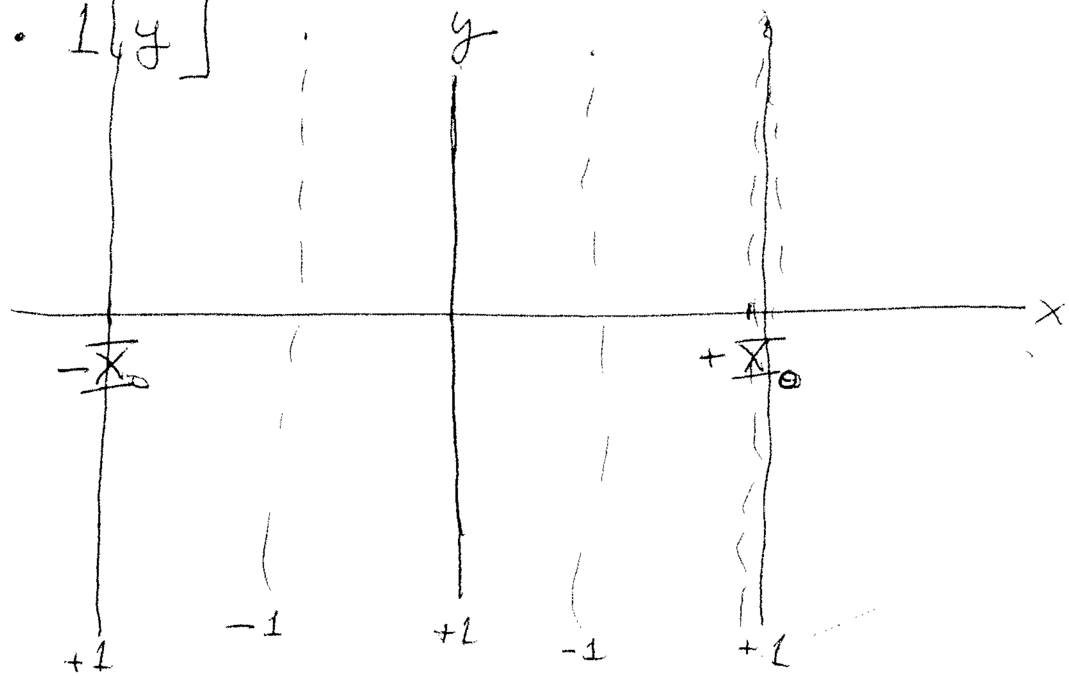
$$0[x,y] = 0[x]\cdot 1[y] = 0[x,y]$$

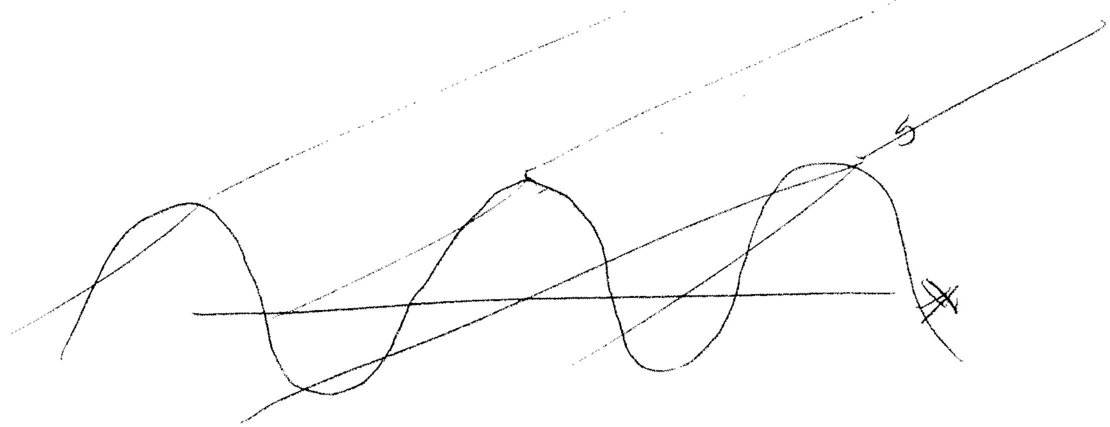
$$\mathrm{SGN}[x]\cdot 1[y]$$

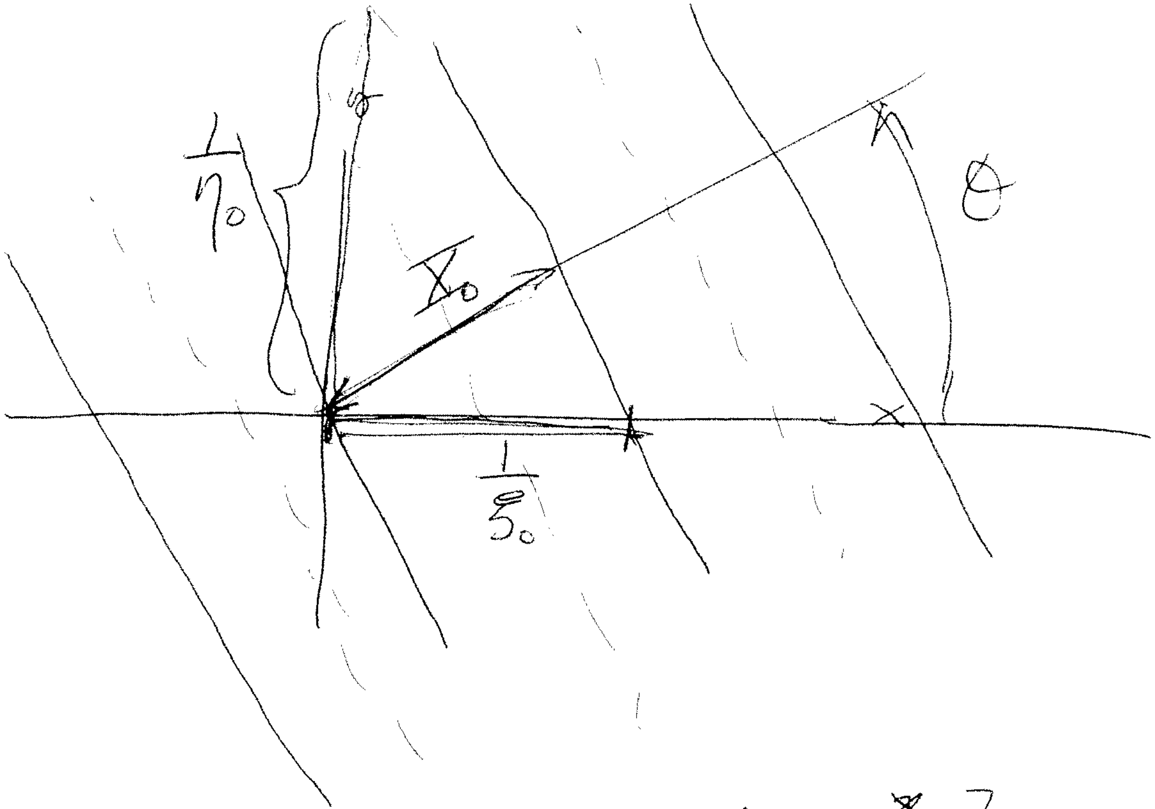




$$\cos[2\pi \xi_0 x] \cdot 1[y]$$





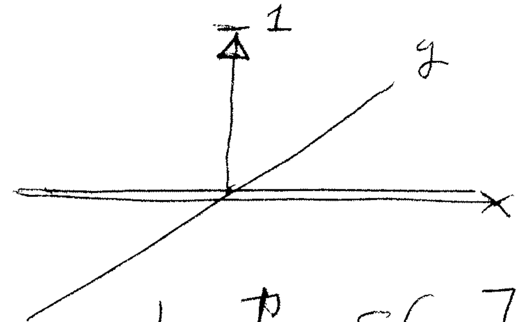


ROTATE $\cos\left[2\pi \frac{x}{X_0}\right] \cdot 1(y)$

$$\cos\left[2\pi\left(\xi_0 x + \eta_0 y\right)\right]$$

2-D Dirac Delta Function

$$\delta[x]\cdot\delta(y] \equiv \delta[x,y]$$



$$\delta\left[\frac{x}{2}\right]\cdot\delta\left[\frac{y}{3}\right] \equiv |2|\,\delta[x]\cdot|3|\cdot\delta(y]$$

$$= 6\;\delta[x,y]$$

$$\delta[x] = \int_{-\infty}^{+\infty} e^{+2\pi i \xi x}\, d\xi$$

$$\delta[y] = \int_{-\infty}^{+\infty} e^{+2\pi i \eta y}\, d\eta$$

$$\delta(x,y) = \iint_{-\infty}^{+\infty} \underbrace{e^{+2\pi i(\xi x+\eta y)}}_{\cos(2\pi(\xi x+\eta y)) + i\sin(2\pi(\xi x+\eta y))} d\xi\, d\eta$$

$1(x,y)$