1051 - 716 FOURIER METHODS

http://www.cis.rit.edu/class/simg716

HW – 30%
EXAM – 30%
FINAL – 40%

DON'T FALL BEHIND!

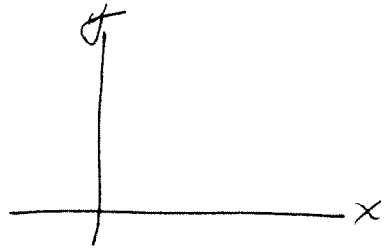
SIGNALSHOW www.signalshow.com

www.DosBox.com

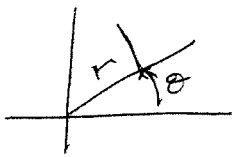
MATHEMATICAL DESCRIPTION OF IMAGING SYSTEMS

FOURIER MATH ⇒ REPRESENT DATA IN "NEW" COORDINATES

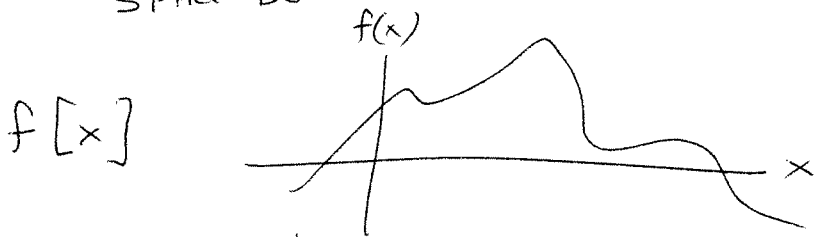
[x, y] CARTESIAN



(r, θ)

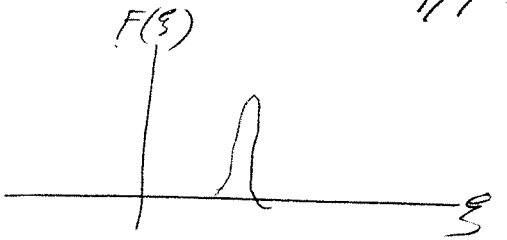


SPACE DOMAIN



$\xrightarrow{\mathcal{F}_1}$

FREQUENCY DOMAIN



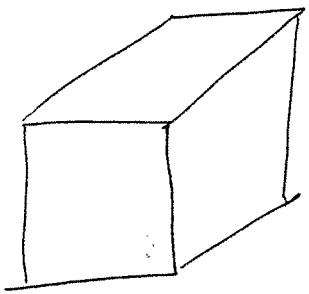
$(\propto \frac{1}{x})$

$f[x,y] \xrightarrow{\mathcal{F}_2} F[\xi,\eta]$

"xi" "eta"

psi ψ

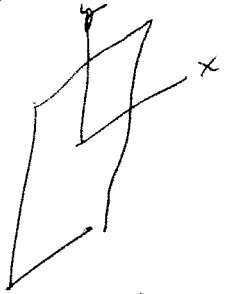
SYSTEMS INPUT



$f[x,y,z,t,\lambda]$

SYSTEM $\mathcal{S}$

OUTPUT



$g[x,y,z,\ldots]$

$g[x,y,\lambda]$

$g[x,y,\lambda_1,\lambda_2,\lambda_3]$

$\mathcal{S}\{f(x,y,\ldots]\} = g[x,y]$

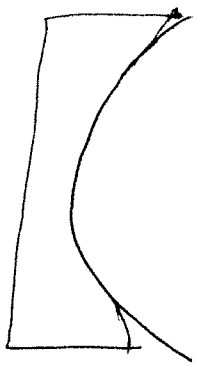
TASKS

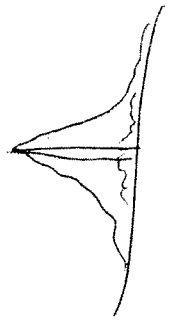
(1) DIRECT: FIND g FROM $\mathcal{A}$ & $f(x,y,f\cdot]$

(2) INVERSE: FIND f FROM $\mathcal{A}$ & g $\quad f[x,y,z]$; $\mathcal{A}$, $g(x,y)$

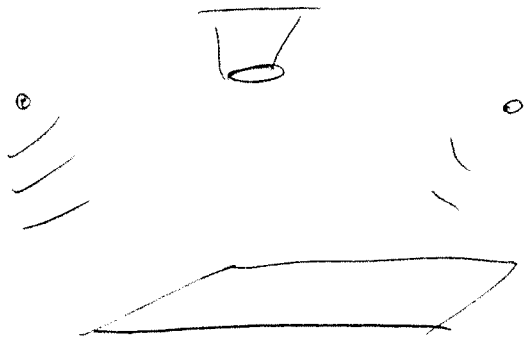
(3) SYSTEM ANALYSIS FIND $\mathcal{A}$ FROM f & g

COSTAR





REMOTE SENSING



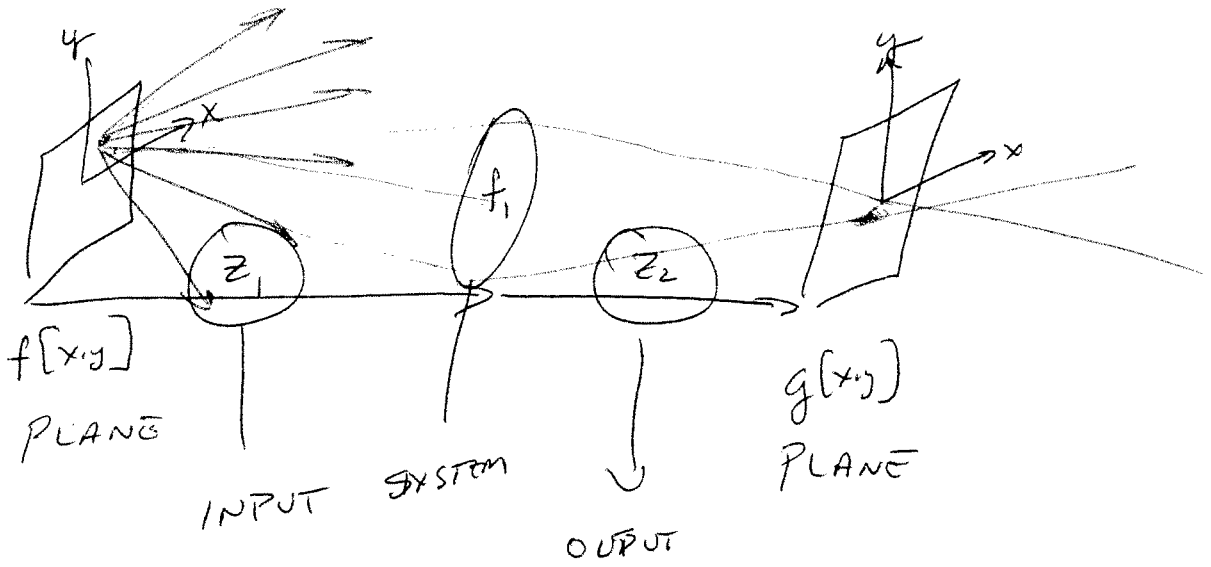
OPTICS — MODELS

SIMPLE —

(1) RAY

(2) WAVE

(3) QUANTUM (SENSOR)

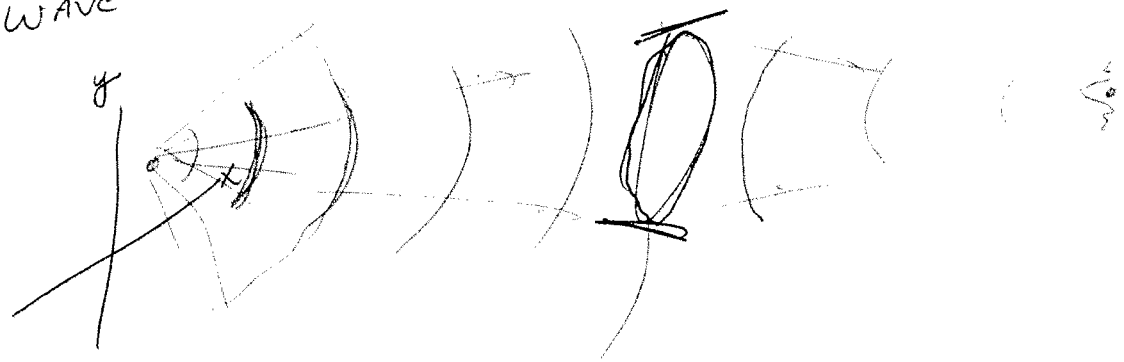


RAY OPTICS IMAGING EQUATION

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \Rightarrow f = \left(\frac{1}{z_1} + \frac{1}{z_2}\right)^{-1}$$ SYSTEM ANALYSIS

$$\begin{cases} z_2 = \left(\frac{1}{f} - \frac{1}{z_1}\right)^{-1} & \text{DIRECT} \\ z_1 = \left(\frac{1}{f} - \frac{1}{z_2}\right)^{-1} & \text{INVERSE} \end{cases}$$

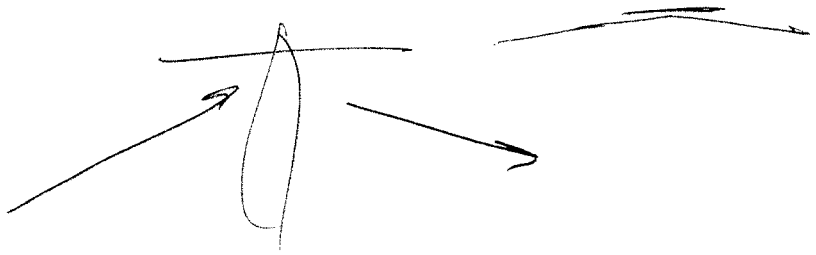
WAVE OPTICS



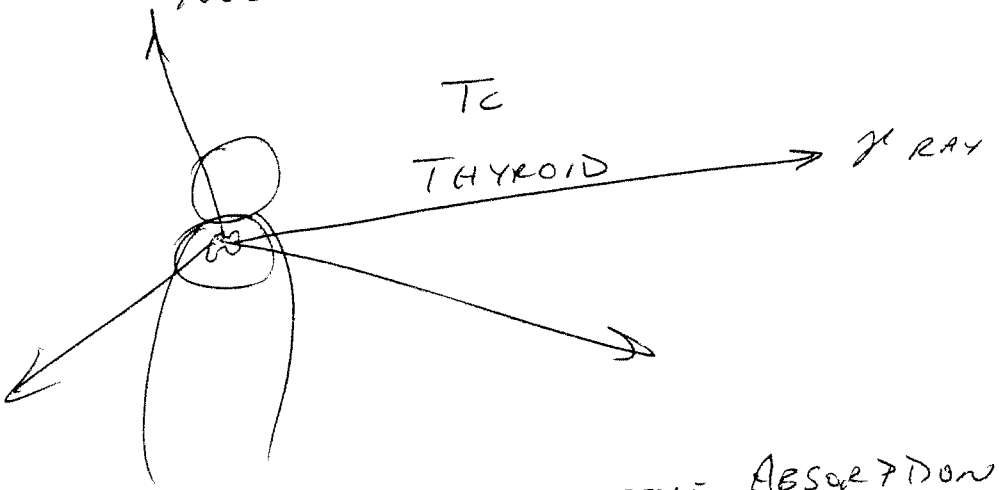
DIFFRACTION

MEDICAL IMAGING

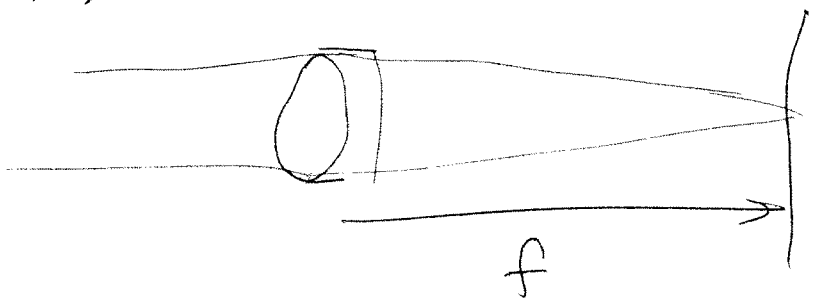
RADIOGRAPHY - X-RAY IMAGING



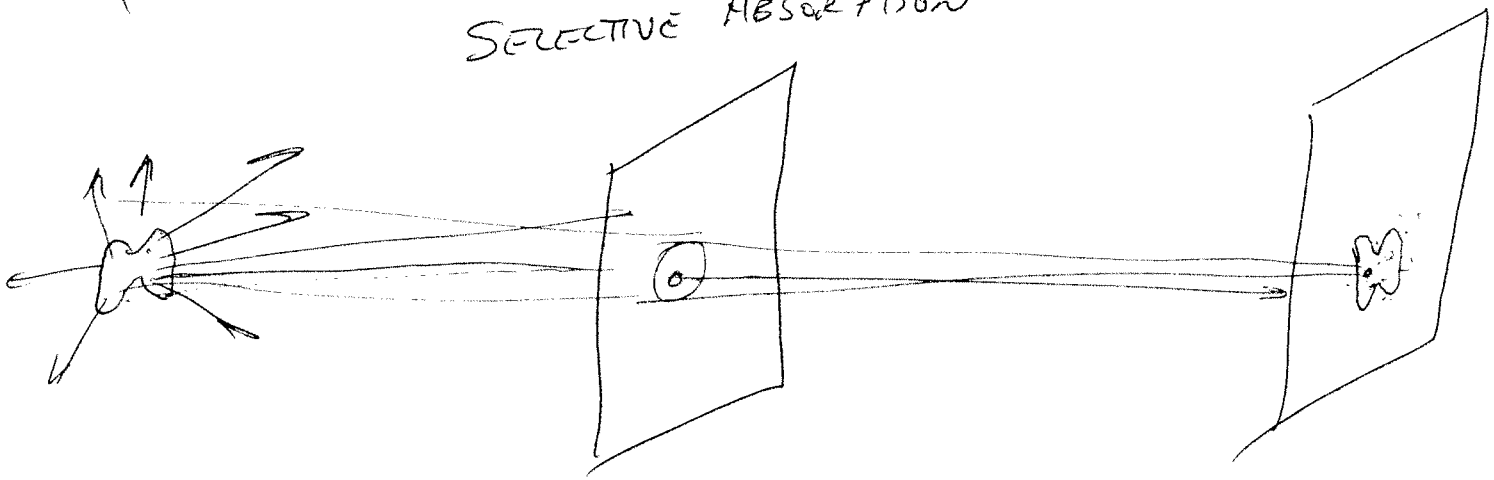
NUCLEAR MEDICINE



GAMMA RAY

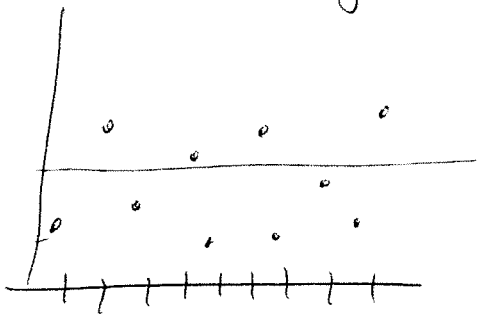


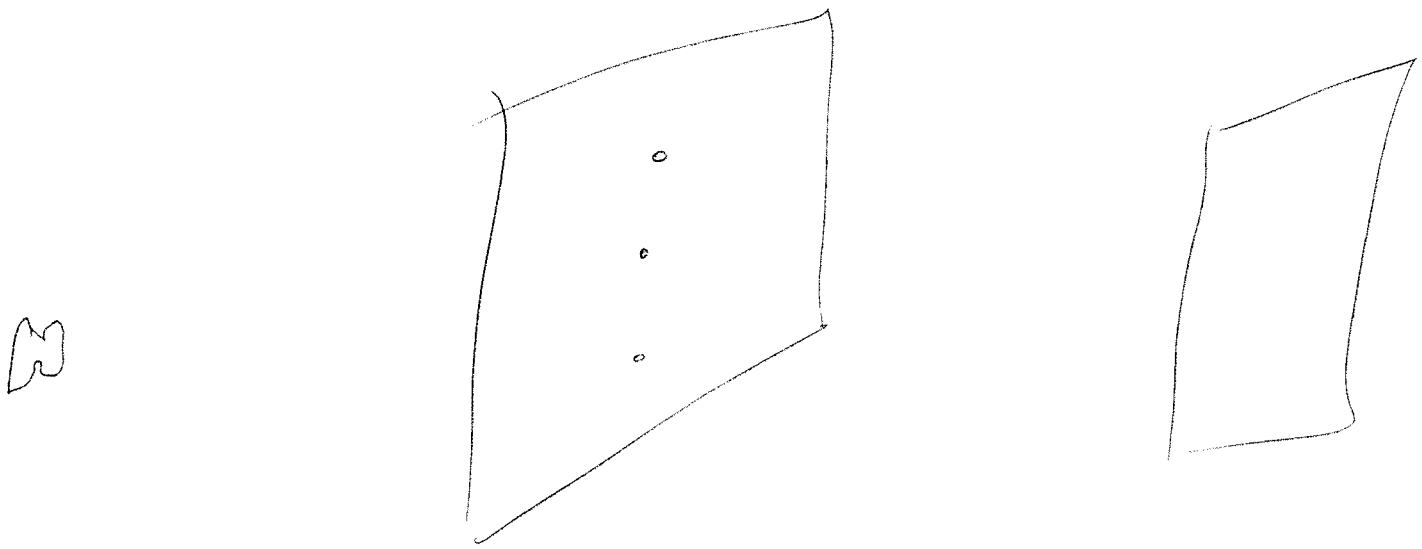
SELECTIVE ABSORPTION



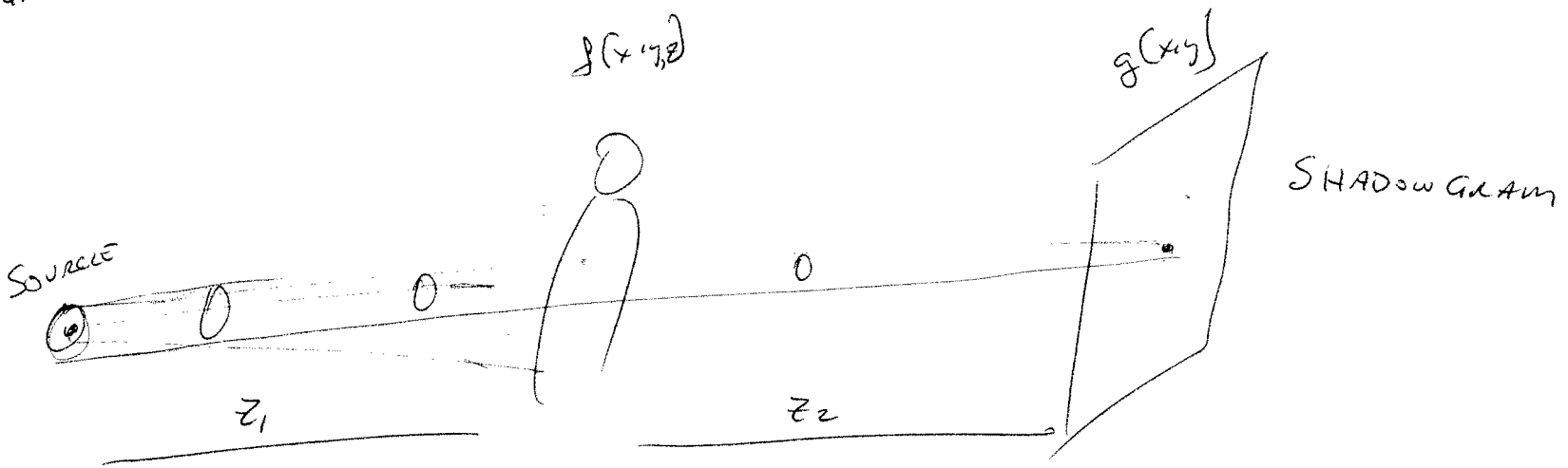
STATISTICAL UNCERTAINTY

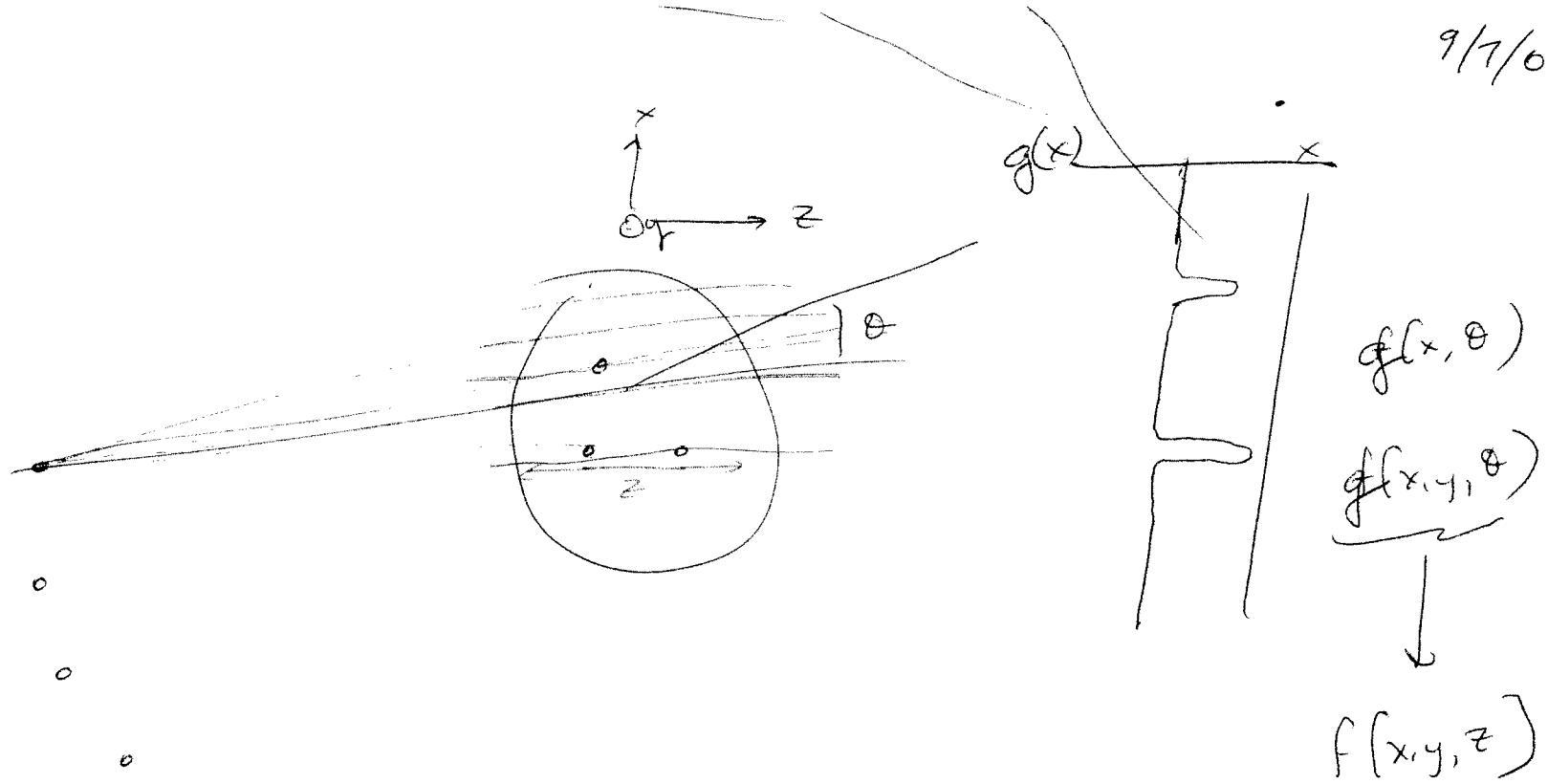
SNR, $\frac{\mu}{\sigma} \propto \sqrt{N}$





RADIOGRAPHY

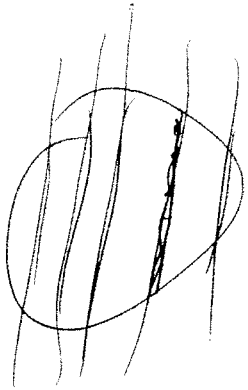




CAT

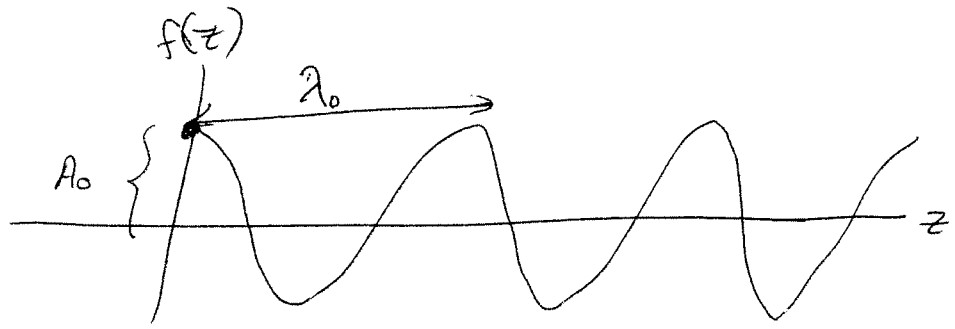
CT COMPUTED TOMOGRAPHY — RADON TRANSFORM 1917

MRI

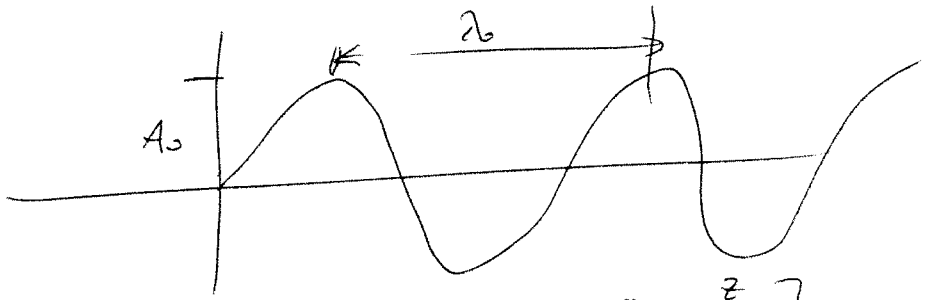


COMPLEX NUMBERS → FUNCTIONS

WAVES



$$f(z) = A_0 \cos\left[2\pi \frac{z}{\lambda_0}\right]$$



$$f[z] = A_0 \sin\left[2\pi \frac{z}{\lambda_0}\right] = A_0 \cos\left[2\pi \frac{z}{\lambda_0} - \frac{\pi}{2}\right]$$

INITIAL PHASE

PHASE ANGLE

GENERAL SINUSOID $A_0 \cos\left[2\pi \frac{z}{\lambda_0} - \varphi_0\right]$

AMPLITUDE ↑ WAVELENGTH ↑ INITIAL PHASE

ϕ_0 φ_0 "PHI"

$\Phi[z; \lambda]$

COMPLEX NUMBERS

$$z = x + iy$$

REAL NUMBERS

$$= a + ib$$

$i \equiv \sqrt{-1}$

a = REAL PART OF z ; $Re\{z\}$ $\mathcal{R}\{z\}$

b = IMAGINARY PART OF z $Im\{z\}$ $\mathcal{I}\{z\}$

ADDITION

~~SUBSTRACTION~~

SUBTRACTION

MULTIPLICATION / DIVISION

RECIPROCAL

$$z_1 \pm z_2 = (a_1 \pm a_2) + i(b_1 \pm b_2)$$

$$z_1 \cdot z_2 = (a_1 + ib_1)\cdot(a_2 + ib_2) = a_1 a_2 + i b_1 a_2 + i b_2 a_1 + i^2 b_1 b_2$$

$$= \underbrace{(a_1 a_2 - b_1 b_2)}_{\text{REAL}} + i\underbrace{(a_1 b_2 + a_2 b_1)}_{\text{IMAG}}$$

$i = \sqrt{-1}$

$i^2 = -1$

$$\frac{1}{z_2} = \frac{1}{a_2 + ib_2} \neq \frac{1}{a_2} + i\frac{1}{b_2}$$

IF $z_2 = 0 + i0 \Rightarrow \frac{1}{z_2}$ IS NOT DEFINED

$a_2 - ib_2 \equiv (z_2)^*$

$$\frac{1}{z_2} = \frac{1}{a_2 + ib_2} \cdot \underbrace{\frac{a_2 - ib_2}{a_2 - ib_2}}_{1} = \frac{1}{z_2}\cdot\frac{z_2^*}{z_2^*}$$

$$\frac{1}{z_2} = \frac{a_2 - ib_2}{(a_2+ib_2)(a_2-ib_2)} = \frac{a_2 - ib_2}{a_2^2 + b_2^2}$$

$$z_2 \cdot z_2^* = (a_2^2 + b_2^2) + i(0)$$

SQUARED MAGNITUDE

$$Re\left\{\frac{1}{z_2}\right\} = \frac{a_2}{a_2^2 + b_2^2}$$

$$Im\left\{\frac{1}{z_2}\right\} = -\frac{b_2}{a_2^2 + b_2^2}$$

$$\frac{z_1}{z_2} = \frac{(a_1+ib_1)(a_2-ib_2)}{(a_2^2 + b_2^2)} = \frac{(a_1a_2 + b_1b_2) + i(a_2b_1 - a_1b_2)}{a_2^2 + b_2^2}$$