SIMG-714 Final Exam Review Topics
Spring, 2001

Information Measure

1. Information Measure

   (a) Entropy, uncertainty and self-information

   (b) Partial information–how much information about an event X_i is carried by a message A?

   (c) May the information about X_i provided by A be negative?

   (d) On the average, information is never negative.

   (e) When can average information provided by a message be zero?

   (f) When is the entropy of a source maximum?

Source Coding

2. Source Coding

   (a) What is an instantaneous code?

   (b) What is a uniquely decodable code?

   (c) What are the conditions for existence? (Kraft and McMillan)

   (d) What are the properties of the digital stream for an efficient code?

   (e) How does the number of digits per symbol relate to the source entropy?

   (f) Huffman coding–methods and properties.

Mutual Information

3. Mutual Information

   (a) Definition of $I(x_i; y_j)$? Show that it is symmetric.

   (b) $I(X;Y) \geq 0$. What are the conditions for $I(X;Y) = 0$ and why do they make sense?

   (c) $H(X,Y)$, $H(X|Y)$,$H(Y|X)$,$H(X)$, $H(Y)$, $I(X;Y)$ definitions and relationships.

   (d) General concept of channel capacity.

   (e) Channels uniform from input, output and both.

   (f) Binary symmetric channel and binary erasure channel

   (g) Time-discrete additive noise channel.

   (h) Under what conditions is it true that $H(Y) = \frac{1}{2}\log(2\pi e\sigma_y^2)$.

(i) When does $C = \frac{1}{2}\ln\left(1 + \frac{\sigma_x^2}{\sigma_z^2}\right)$?

(j) For what kind of situation is it always true that $I(X; Z|Y) = 0$? Why?

Linear Block Codes

4. A binary linear block code uses takes in k information digits and produces codewords of length $n > k$.

   (a) Every codeword can be generated by multiplying a matrix $\mathbf{G}$ by a vector $\mathbf{m} = [m_0, m_1, \ldots, m_{k-1}]$. That is , $\mathbf{c} = \mathbf{mG}$ is a codeword. How many different codewords are in the code?

   (b) Show that every row of $\mathbf{G}$ is a codeword.

   (c) Show that $\mathbf{0}$ is in every linear block code. By the way, what is the difference between $\mathbf{0}$ and 0?

   (d) Prove that the code created in this way is linear.

   (e) Find a set of basis vectors that spans the code.

   (f) Suppose that $\mathbf{h}_1$ is an n digit binary vector such that $\mathbf{ch}_1^T = 0$ for every codeword, $\mathbf{c}$. Show that $\mathbf{Gh}_1^T = \mathbf{0}$.

   (g) How large is the set of linearly independent vectors that are orthogonal to the code space?

   (h) What is a systematic code?

   (i) Is it true that every linear block code is equivalent to a systematic code?

   (j) Let $\mathbf{G} = [\mathbf{I}_k|\mathbf{P}]$. Then $\mathbf{c} = \mathbf{mG}$ has a particular structure. What is it?

   (k) Show that $\mathbf{H} = [\mathbf{P}^T|\mathbf{I}_{n-k}]$ and that $\mathbf{GH}^T = \mathbf{0}$.

   (l) What is a syndrome and why is it important.

   (m) Suppose that $\mathbf{e}_1$ and $\mathbf{e}_2$ are error patterns with the same syndrome. What can you say about $\mathbf{e}_1 + \mathbf{e}_2$?

   (n) How large is the set of all error patterns that have the same syndrome?

   (o) How can you compute the syndrome from a received n digit word $\mathbf{v}$ that may or may not contain errors?

   (p) Explain why the syndrome for a received word $\mathbf{v} = \mathbf{c}_i + \mathbf{e}$ depends only on the error pattern and not the codeword.

   (q) If you have a syndrome $\mathbf{s}$, how do you decide which error pattern was the most likely to have caused it?

5. Let A be a standard array for a linear block code.

(a) How can a generator matrix be determined?

(b) Is the generator matrix unique?

(c) How can a decoding table be constructed?

(d) Suppose you do not like the decoding table because each syndrome is not associated with the minimum weight error pattern. How can the standard array be modified to rectify this problem?

(e) Given a received n digit word $\mathbf{v}= \mathbf{c_i} + \mathbf{e}$, how do you use A to find the most probable codeword and most probable error pattern?

(f) What are the dimensions of a standard array for a Hamming code?

6. Let $A(z)$ be a weight distribution polynomial for a linear block code.

   (a) What do the coefficients represent?

   (b) How would you go about computing the probability of an undetectable error pattern for this code?

   (c) If you are given $A(z)$, can you determine the smallest number of errors that must occur to be undetectable?

   (d) What does $A(1)$ always equal?

   (e) What does $A(0)$ always equal?

Transform Coding and Image Compression

7. Let $F(x, y)$ represent an image array and $G(u, v)$ be its transform produced by

$$G(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} S(x,y,u,v) F(x,y)$$

   What properties must be possessed by the kernel S for the transformation to be separable? Why is separability important?

8. Suppose that $S(x, y, u, v) = S_1(x, u) S_2(y, v)$. Show how to construct matrices $\mathbf{P}$ and $\mathbf{Q}$ so that $\mathbf{G} = \mathbf{PFQ}$.

9. The Discrete Cosine Transform has the equation

$$G(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} F(x,y) \cos\frac{(2x+1)u\pi}{2N} \cos\frac{(2y+1)v\pi}{2N}$$

   where $\alpha(0) = \frac{1}{\sqrt{N}}$ and $\alpha(u) = \frac{2}{\sqrt{N}}$ for $u > 0$. Construct the matrices $\mathbf{P}$ and $\mathbf{Q}$ for the case $N = 4$. What can you say about the product $\mathbf{P}^T\mathbf{P}$?

10. What are the properties of the DCT that make it attractive for image compression?

11. At what step in JPEG compression does the "lossy" compression take place?

12. How can you regulate the quality vs compression tradeoff in JPEG?

13. What is the zig-zag scanning of the quantized array in JPEG good for?

14. What is the "DCT progressive mode", why is it used, and how does it work?

15. Why does JPEG use image blocks?

16. About how many bits per pixel are needed with JPEG to produce excellent quality?

17. How does JPEG handle color images?

18. Where was Gregory K. Wallace working at the time he wrote the JPEG paper?