

SIMG-713

Homework 4

1. Let S be the number of successes in 100 Bernoulli trials with $p = 0.4$ equal to the probability of success on any one trial. Find the probability that the number of successes exceeds 45.
2. Consider a sequence of Bernoulli trials with $p = 0.5$ as the probability of success on any single trial. Determine the number n of trials that you must conduct such the probability $P[S_n/n > 0.51] \leq 0.01$. Hint: Assume that n is large enough so that the binomial distribution can be approximated by the normal distribution. You can then use the error function or other tables for the normal distribution.
3. A sensor array with 1000 elements is struck by 1000 photons. Estimate the probability that a given element is struck by at least three photons. Ignore any interactions between elements.
4. A book contains 500 pages and contains on the average 1 misprint every 10 pages. Estimate the probability that a random selection of 5 pages will contain 2 misprints. Does it matter whether the five pages are contiguous?
5. Raindrops strike an area of 10 square centimeters at a rate of 1 drop per second. What is the probability that a given area of 1 square centimeter will not be struck during a time interval of length 1 minute?
6. An array of 100 detectors is placed in a photon stream. The average number of photons arriving at a detector is $q = 5$. What is the standard deviation in the number of photons per detector?
7. A certain photomultiplier emits N electrons when it is struck by one photon. The number N is a random variable with the probability distribution given by the table below.

N	0	1	2	3
$P(N)$.1	.2	.3	.4

The detector is actually struck by X photons per second, with $E[X] = 4$, which leads to the production of Y electrons. What is $E[Y]$, the expected number of electron emissions per second?

8. What is the variance of Y for the above problem at $E[X] = 4$?
9. Suppose that the brightness of the input photon stream is changed to $E[X] = 5$. Find the new value of $E[Y]$ and use it to estimate “gain” g of the system. That is, what is the rate of change of $E[Y]$ with a change in $E[X]$?

10. Compute the ratio

$$r = \frac{g^2 \text{var}[X]}{\text{var}[Y]}$$

using the results of problems 8 and 9. Would you expect this ratio to be larger or smaller than 1.0. Explain.